13

Package-Based Description Logics

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Summary. We present the syntax and semantics of a family of modular ontology languages, Package-based Description Logics (P-DL), to support context- specific reuse of knowledge from multiple ontology modules. In particular, we describe a P-DL $\mathcal{SHOIQP}$ that allows the *importing* of concept, role and nominal names between multiple ontology modules (each of which can be viewed as a $\mathcal{SHOIQ}$ ontology). $\mathcal{SHOIQP}$ supports contextualized interpretation, i.e., interpretation from the point of view of a specific package. We establish the necessary and sufficient conditions on domain relations (i.e., the relations between individuals in different local domains) that need to hold in order to preserve the unsatisfiability of concept formulae, monotonicity of inference, transitive reuse of knowledge across modules.

13.1 Introduction

The success of the world wide web can be partially attributed to the *network effect*: The absence of central control on the content and the organization of the web allows thousands of independent actors to contribute resources (web pages) that are interlinked to form the web. Ongoing efforts to extend the current web into a *semantic web* are aimed at enriching the web with machine interpretable content and interoperable resources and services [7]. Realizing the full potential of the semantic web requires the large-scale adoption and use of ontology-based approaches to sharing of information and resources. Constructing large ontologies typically requires collaboration among multiple individuals or groups with expertise in specific areas, with each participant contributing only a part of the ontology. Therefore, instead of a single, centralized ontology, in most application domains it is natural to have multiple distributed ontologies covering parts of the domain. Such ontologies represent the *local* knowledge of the ontology designers, i.e., knowledge that is applicable in a *context*. Because no single ontology can meet the needs of all users under every conceivable scenario, there is an urgent need for theoretically sound, yet practical,

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approaches that allow knowledge from multiple autonomously developed ontologies to be adapted and reused in user, context, or application-specific scenarios.

Ontologies on the semantic web need to satisfy two apparently conflicting objectives [9]:

- *Sharing* and *reuse* of knowledge across autonomously developed ontologies. An ontology may reuse another ontology by direct *importing* of selected terms in the other ontology (e.g., by referring to their URLs), or by using *mappings* between ontologies.
- The *contextuality* of knowledge or accommodation of the *local points of view*. For example, an assertion of the form "everything has the property that..." is usually made within an implicit local context which is often omitted from the statement. In fact, such a statement should be understood as "everything *in this domain* has the property that...". However, when reusing an existing ontology, the contextual nature of assertions is often neglected, leading to unintended inferences.

OWL adopts an importing mechanism to support integration of ontology modules. However, the importing mechanism in OWL, implemented by the `owl:imports` construct, in its current form, suffers from several serious drawbacks: (a) It directly introduces both terms and axioms of the imported ontologies into the importing ontology, and thus fails to support contextual reuse; (b) It provides no support for partial reuse of an ontology module.

Consequently, there have been several efforts aimed at developing formalisms that allow *reuse* of knowledge from multiple ontologies via *contextualized interpretations* in multiple local domains instead of a single shared global interpretation domain. Contextualized reuse of knowledge requires the interactions between local interpretations to be controlled. Examples of such modular ontology languages include: Distributed Description Logics (DDL) [8], $\mathcal{E}$-Connections [16] and Semantic Importing [20].

An alternative approach to knowledge reuse is based on the notion of *conservative extension* [12, 13, 14, 15], which allows ontology modules to be interpreted using standard semantics by requiring that they share the same global interpretation domain. To avoid undesired effects from combining ontology modules, this approach requires that such a combination be a conservative extension of component modules. More precisely, if O is the union of a set of ontology modules $\{O_1, ..., O_n\}$, then we say O is a conservative extension of O_i if $O \models \alpha \Leftrightarrow O_i \models \alpha$, for any α in the language of O_i. This guarantees that combining knowledge from several ontology modules does not alter the consequences of knowledge contained in any component module. Thus, a combination of ontology modules cannot induce a new concept inclusion relation between concepts expressible in any of the component modules.

Current approaches to knowledge reuse have several limitations. To preserve contextuality, existing modular ontology languages offer only limited ways to connect ontology modules and, hence, limited ability to reuse knowledge across modules. For instance, DDL does not allow concept construction using foreign roles or concepts. $\mathcal{E}$-Connections, on the other hand, does not allow concept subsumptions across ontology modules or the use of foreign roles. Finally, Semantic Importing, in its current form, only allows each component module to be in $\mathcal{ALC}$. None of the existing approaches supports knowledge reuse in a setting where each ontology module uses a representation language that is as expressive as OWL-DL, i.e., $\mathcal{SHOIN}(D)$.

Furthermore, some of the existing modular ontology languages suffer from reasoning difficulties that can be traced back to the absence of natural ways to restrict the relations between individuals in different local domains. For example, DDL does not support the transitivity of inter-module concept subsumptions (known as *bridge rules*) in general. Moreover, in DDL a concept that is declared as being more specific than two disjoint concepts in another module may still be satisfiable (the inter-module satisfiability problem) [3, 16]. Undisciplined use of generalized links in $\mathcal{E}$-Connections has also been shown to lead to reasoning difficulties [2].

Conservative extensions [13, 14, 15], in their current form, require a single global interpretation domain and, consequently, prevent different modules from interpreting axioms within their own local contexts. Hence, the designers of different ontology modules have to anticipate all possible contexts in which knowledge from a specific module might be reused. As a result, several modeling scenarios that would, otherwise, be quite useful in practice, such as the refinement of relations between existing concepts in an ontology module and the general reuse of nominals [19], are precluded.

Against this background, this chapter, building on previous work of a majority of the authors [3], develops a formalism that can support *contextual* reuse of knowledge from multiple ontology modules. The resulting modular ontology language, Package-based Description Logic (P-DL) $\mathcal{SHOIQP}$:

- Allows each ontology module to use a subset of $\mathcal{SHOIQ}$ [17], i.e., $\mathcal{ALC}$ augmented with transitive roles, role inclusion, role inversion, qualified number restriction and nominal concepts and, hence, covers a significant fragment of OWL-DL.
- Supports more flexible modeling scenarios than those supported by existing approaches through a mechanism of *semantic importing* of names (including concept, role and nominal names) across ontology modules[1].
- Contextualizes the interpretation of reused knowledge. Locality of axioms in ontology modules is obtained "for free" by its *contextualized semantics*, thereby freeing ontology engineers from the burden of ensuring the reusability of an ontology module in contexts that are hard to foresee when constructing the module. A natural consequence of contextualized interpretation is that inferences are always drawn *from the point of view* of a *witness* module. Thus, different modules might infer different consequences, based on the knowledge that they import from other modules.

[1] Note that importing in OWL, implemented by the `owl:imports` is essentially syntactic in nature. The difference between syntactic importing and semantic importing is best illustrated by an analogy with the writing of scientific articles: Knowledge reuse via `owl:imports` analogous to *cut and paste* from a source article; In contrast, semantic importing is akin to knowledge reuse by means of *citation* of source article.

- Ensures that the results of reasoning are always the same as those obtained by a standard reasoner over an integrated ontology resulting from combining the relevant knowledge in a context-specific manner. Thus, unlike in the case of DDL and Semantic Importing of Pan et al., P-DL ensures the *monotonicity* of inference in the distributed setting.
- Avoids several of the known reasoning difficulties of the existing approaches, e.g., lack of support for transitive reusability and nonpreservation of concept unsatisfiability.

13.2 Semantic Importing

This section introduces the syntax and semantics of the proposed language $\mathcal{SHOIQP}$. We will use a simple example shown in Figure 13.1 to illustrate some of the basic features of the P-DL syntax.

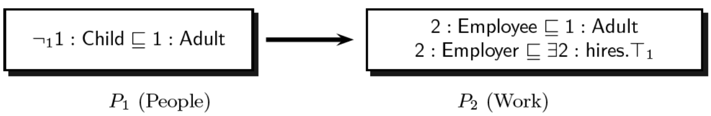


Fig. 13.1. Semantic Importing

13.2.1 Syntax

Packages

Informally, a package in $\mathcal{SHOIQP}$ can be viewed as a $\mathcal{SHOIQ}$ TBox and RBox. For example, in Figure 13.1 there are two packages, package P_1 describes the domain of People and P_2 describes the domain of Work.

We define the *signature* $\mathsf{Sig}(P_i)$ of a package P_i as the set of names used in P_i. $\mathsf{Sig}(P_i)$ is the disjoint union of the set of concept names NC_i, the set of role names NR_i and the set of nominal names NI_i used in package P_i. The set of roles in P_i is defined as $\overline{\mathsf{NR}}_i = \mathsf{NR}_i \cup \{R^- | R \in \mathsf{NR}_i\}$ where R^- is the *inverse* of the role name R.

The signature $\mathsf{Sig}(P_i)$ of package P_i is divided into two disjoint parts: its local signature $\mathsf{Loc}(P_i)$ and its external signature $\mathsf{Ext}(P_i)$. Thus, in the example shown in Figure 13.1, $\mathsf{Sig}(P_2) = \{\mathsf{Employee}, \mathsf{Adult}, \mathsf{Employer}, \mathsf{hires}\}$; $\mathsf{Loc}(P_2) = \{\mathsf{Employee}, \mathsf{Employer}, \mathsf{hires}\}$; and $\mathsf{Ext}(P_2) = \{\mathsf{Adult}\}$.

For all $t \in \mathsf{Loc}(P_i)$, P_i (and only P_i) is the *home package* of t, denoted by $P_i = \mathsf{Home}(t)$, and t is called an i-*name* (more specifically, an i-*concept name*, an i-*role name*, or an i-*nominal name*). We will use "$i : X$" to denote an i-name X and may drop the prefix when it is clear from the context. We use i-*role* to refer to an i-role name or its *inverse*. In the example shown in Figure 13.1, the home package of the terms Child and Adult is P_1 (People); and that of Employee, Employer and hires is P_2 (Work).

A role name $R \in \mathsf{NR}_i$ may be declared to be *transitive* in P_i using an axiom $\mathsf{Trans}_i(R)$. If R is declared transitive, R^- is also said to be *transitive*. We use $\mathsf{Tr}_i(R)$ to denote a role R being transitive in P_i.

A *role inclusion* axiom in P_i is an expression of the form $R \sqsubseteq S$, where R and S are i-roles. The *role hierarchy* for P_i is the set of all role inclusion axioms in P_i. The RBox $\mathcal{R}_i$ consists of the role hierarchy $\mathbf{R}_i$ for P_i and the set of role transitivity declarations $\mathsf{Trans}_i(R)$. For a role hierarchy $\mathbf{R}_i$, if $R \sqsubseteq S \in \mathbf{R}_i$, then R is called a *sub-role* of S and S is called a *super-role* of R *w.r.t.* $\mathbf{R}_i$. An i-role is called *locally simple* if it neither transitive nor has any transitive sub-role in P_i.

The set of $\mathcal{SHOIQP}$ concepts in P_i is defined inductively by the following grammar:

$$C := A|o|\neg_k C|C \sqcap C|C \sqcup C|\forall R.C|\exists R.C|(\leq nS.C)|(\geq nS.C)$$

where $A \in \mathsf{NC}_i$, $o \in \mathsf{NI}_i$, n is a non-negative integer, $R \in \overline{\mathsf{NR}}_i$, and $S \in \overline{\mathsf{NR}}_i$ is a locally simple role; $\neg_k C$ denotes the *contextualized negation* of concept *C w.r.t.* P_k. For any *k* and k-concept name C, $\top_k = \neg_k C \sqcup C$, and $\bot = \neg_k C \sqcap C$. Thus, there is no universal top ($\top$) concept or global negation ($\neg$). Instead, we have for each package P_k, a contextualized top $\top_k$ and a contextualized negation $\neg_k$. This allows a logical formula in P-DL (including $\mathcal{SHOIQP}$) to be interpreted within the context of a specific package. Thus, in the example shown in Figure 13.1, $\neg_1 1 : \mathsf{Child}$ in P_1 describes only the individuals *in the domain of People* that are not not children (that is, *not* $1 : \mathsf{Child}$).

A *general concept inclusion* (GCI) *axiom* in P_i is an expression of the form $C \sqsubseteq D$, where C, D are concepts in P_i. The TBox $\mathcal{T}_i$ of P_i is the set of GCIs in P_i. Thus, formally, a *package* P_i is a pair $P_i := \langle \mathcal{T}_i, \mathcal{R}_i \rangle$. A $\mathcal{SHOIQP}$ ontology Σ is a set of packages $\{P_i\}$. We assume that every name used in a $\mathcal{SHOIQP}$ ontology Σ has a home package in Σ.

Semantic Importing between Packages

If a concept, role or nominal name $t \in \mathsf{Loc}(P_j) \cap \mathsf{Ext}(P_i)$, $i \neq j$, we say that P_i *imports* t and denote it as $P_j \xrightarrow{t} P_i$. We require that transitivity of roles be preserved under importing. Thus, if $P_j \xrightarrow{R} P_i$ where R is a j-role name, then $\mathsf{Trans}_i(R)$ iff $\mathsf{Trans}_j(R)$. If any local name of P_j is imported into P_i, we say that P_i imports P_j and denote it by $P_j \mapsto P_i$. In the example shown in Figure 13.1, P_2 imports P_1.

The *importing transitive closure* of a package P_i, denoted by P_i^+, is the set of all packages that are directly or indirectly imported by P_i. That is, P_i^+ is the smallest subset of $\{P_i\}$, such that

- $\forall j \neq i, P_j \mapsto P_i \Rightarrow P_j \in P_i^+$
- $\forall k \neq j \neq i, (P_k \mapsto P_j) \wedge (P_j \in P_i^+) \Rightarrow P_k \in P_i^+$

Let $P_i^* = \{P_i\} \cup P_i^+$. A $\mathcal{SHOIQP}$ ontology $\Sigma = \{P_i\}$ has an *acyclic importing relation* if, for all i, $P_i \notin P_i^+$; otherwise, it has a *cyclic importing relation*. The importing relation in the example in Figure 13.1 is acyclic.

We denote a Package-based Description Logic (P-DL) by adding the letter $\mathcal{P}$ to the notation for the corresponding DL. For example, $\mathcal{ALCP}$ is the package extension of the DL $\mathcal{ALC}$. We denote by $\mathcal{P}_\mathcal{C}$ a restricted type of P-DL that only allows importing of concept names. $\mathcal{P}^-$ denotes a P-DL with acyclic importing. In particular, $\mathcal{ALCP}_{\mathcal{C}}^-$ was studied in [1], $\mathcal{ALCP}_{\mathcal{C}}$ was studied in [4] and $\mathcal{SHOIQP}$ was studied in [5]. The example in Figure 13.1 is in $\mathcal{ALCP}_{\mathcal{C}}^-$.

Syntax Restrictions on Semantic Importing

Restrictions on Negations. We require that $\neg_k C$ (hence also $\top_k$) can appear in P_i, $i \neq k$, only if $P_k \mapsto P_i$. Intuitively, this means that k-negation can appear only in P_k or any package that directly imports P_k.

Restrictions on Imported Role Names. We require that *an imported role should not be used in role inclusion axioms*. This restriction is imposed because of two reasons. First, decidability requires that a role that is used in number restrictions be "globally" simple, i.e., that it has no transitive sub-role across any importing chain[2] [18]. In practice, it is useful to restrict the use of imported roles in such a way that a role is globally simple iff it is locally simple. Second, a reduction of $\mathcal{SHOIQP}$ without such a restriction to an integrated ontology may require some features that are beyond the expressivity of $\mathcal{SHOIQ}$, such as role intersection.

$\mathcal{SHOIQP}$ Examples

The semantic importing approach described here can model a broad range of scenarios that can also be modeled using existing approaches.

Example 1. Inter-module concept and role inclusions. *Suppose we have a* people *ontology P_1:*

$$\neg_1 1:\mathsf{Man} \sqsubseteq 1:\mathsf{Woman}$$
$$1:\mathsf{Man} \sqsubseteq 1:\mathsf{People}$$
$$1:\mathsf{Woman} \sqsubseteq 1:\mathsf{People}$$
$$1:\mathsf{Boy} \sqcup 1:\mathsf{Girl} \sqsubseteq 1:\mathsf{Child}$$
$$1:\mathsf{Husband} \sqsubseteq 1:\mathsf{Man} \sqcap \exists 1:\mathsf{marriedTo}.1:\mathsf{Woman}$$

Suppose the Work *ontology P_2 imports some of the knowledge from the people ontology:*

$$2:\mathsf{Employee} \sqsubseteq 1:\mathsf{People} \quad (13.1)$$
$$2:\mathsf{Employer} \equiv \exists 2:\mathsf{hires}.1:\mathsf{People} \quad (13.2)$$
$$1:\mathsf{Child} \sqsubseteq \neg_2 2:\mathsf{Employee} \quad (13.3)$$
$$2:\mathsf{EqualOpportunityEmployer} \sqsubseteq \exists 2:\mathsf{hires}.1:\mathsf{Man} \sqcap \exists 2:\mathsf{hires}.1:\mathsf{Woman} \quad (13.4)$$

Axioms (13.1) *models inter-module concept inclusion. This example also illustrates that the semantic importing approach can realize concept specialization (Axiom* (13.2)*) and generalization (Axiom* (13.3)*).*

[2] This follows from the reduction from $\mathcal{SHOIQP}$ to $\mathcal{SHOIQ}$ given in Section 13.3.

Example 2. Use of foreign roles or foreign concepts to construct local concepts. *Suppose a* marriage *ontology P_3 reuses the people ontology:*

$$(=1\ (1:\mathsf{marriedTo}).(1:\mathsf{Woman})) \sqsubseteq 3:\mathsf{Monogamist} \tag{13.5}$$

$$3:\mathsf{MarriedPerson} \sqsubseteq \forall(1:\mathsf{marriedTo}).(3:\mathsf{MarriedPerson}) \tag{13.6}$$

$$3:\mathsf{NuclearFamily} \sqsubseteq \exists(3:\mathsf{hasMember}).(1:\mathsf{Child}) \tag{13.7}$$

A complex concept in P_3 may be constructed using an imported role (13.6)*, an imported concept* (13.7)*, or both an imported role and an imported concept* (13.5).

Example 3. The use of nominals. *Suppose the work ontology P_2, defined above, is augmented with additional knowledge from a* calendar *ontology P_4, to obtain an augmented work ontology. Suppose P_4 contains the following axiom:*

$$4{:}\mathsf{WeekDay} = \{4{:}Mon, 4{:}Tue, 4{:}Wed, 4{:}Thu, 4{:}Fri\},$$

where the nominals are shown in italic font. Suppose the new version of P_2 contains the following additional axioms:

$$4:Fri \sqsubseteq \exists(2:\mathsf{hasDressingCode}).(2:\mathsf{CasualDress})$$

$$\top_2 \sqsubseteq \exists(2:\mathsf{hasDressingCode}^-).(4:\mathsf{WeekDay})$$

13.2.2 Semantics

A $\mathcal{SHOIQP}$ ontology has *localized semantics* in the sense that each package has its own local interpretation domain. Formally, for a $\mathcal{SHOIQP}$ ontology $\Sigma = \{P_i\}$, a *distributed interpretation* is a tuple $\mathcal{I} = \langle\{\mathcal{I}_i\}, \{r_{ij}\}_{P_i \in P_j^+}\rangle$, where $\mathcal{I}_i$ is a *local interpretation* of package P_i, with (a not necessarily non-empty) domain $\Delta^{\mathcal{I}_i}$, $r_{ij} \subseteq \Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_j}$ is the *(image) domain relation* for the interpretation of the direct or indirect importing relation from P_i to P_j. For convenience, we use $r_{ii} = \mathrm{id}_{\Delta^{\mathcal{I}_i}} := \{(x,x)|x \in \Delta^{\mathcal{I}_i}\}$ to denote the identity mapping in the local domain $\Delta^{\mathcal{I}_i}$. Taking this convention into account, the distributed interpretation $\mathcal{I} = \langle\{\mathcal{I}_i\}, \{r_{ij}\}_{P_i \in P_j^+}\rangle$ may also be denoted by $\mathcal{I} = \langle\{\mathcal{I}_i\}, \{r_{ij}\}_{P_i \in P_j^*}\rangle$.

To facilitate our further discussion of interpretations, the following notational conventions will be used throughout. Given i, j, such that $P_i \in P_j^*$, for every $x \in \Delta^{\mathcal{I}_i}$, $A \subseteq \Delta^{\mathcal{I}_i}$ and $S \subseteq \Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_i}$, define[3] (please see Figure 13.2 and 13.3 for illustration):

$$r_{ij}(A) = \{y \in \Delta^{\mathcal{I}_j} | \exists x \in A, (x,y) \in r_{ij}\}, \qquad \text{(concept image)}$$

$$r_{ij}(S) = r_{ij} \circ S \circ r_{ij}^- \qquad \text{(role image)}$$

$$= \{(z,w) \in \Delta^{\mathcal{I}_j} \times \Delta^{\mathcal{I}_j} | \exists (x,y) \in S, (x,z) \in r_{ij} \wedge (y,w) \in r_{ij}\},$$

$$S(x) = \{y \in \Delta^{\mathcal{I}_i} | (x,y) \in S\} \qquad \text{(successor set)}$$

[3] In this chapter, $f_1 \circ ... \circ f_n$ denotes the composition of n relations $f_1, ..., f_n$, i.e., $(f_1 \circ ... \circ f_n)(x) = f_1(...f_n(x))$.

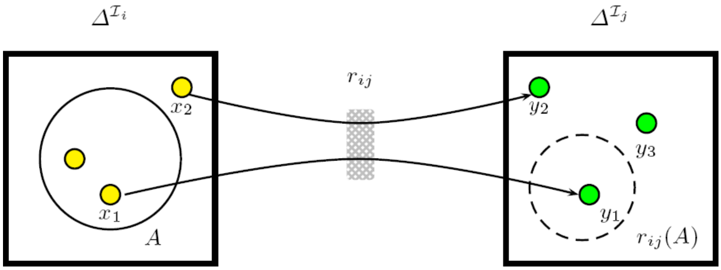


Fig. 13.2. Concept Image

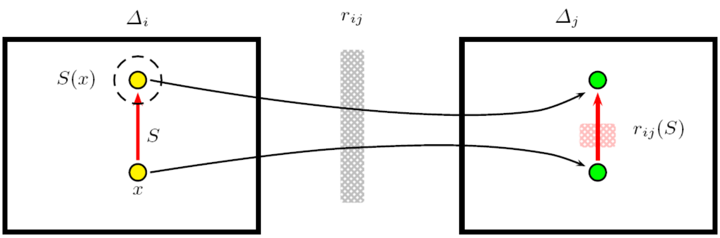


Fig. 13.3. Successor Set and Role Image

Moreover, let ρ be the equivalence relation on $\bigcup_i \Delta^{\mathcal{I}_i}$ generated by the collection of ordered pairs $\bigcup_{P_i \in P_j^*} r_{ij}$. This is the symmetric and transitive closure of the set $\bigcup_{P_i \in P_j^*} r_{ij}$. Define, for every i, j, $\rho_{ij} = \rho \cap (\Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_j})$.

Each of the local interpretations $\mathcal{I}_i = \langle \Delta^{\mathcal{I}_i}, \cdot^{\mathcal{I}_i} \rangle$ consists of a domain $\Delta^{\mathcal{I}_i}$ and an interpretation function $\cdot^{\mathcal{I}_i}$, which maps every concept name to a subset of $\Delta^{\mathcal{I}_i}$, every role name to a subset of $\Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_i}$ and every nominal name to an element in $\Delta^{\mathcal{I}_i}$. We require that the interpretation function $\cdot^{\mathcal{I}}$ satisfies the following equations, where R is a j-role, S is a locally simple j-role, C, D are concepts:

$$
\begin{aligned}
R^{\mathcal{I}_i} &= (R^{\mathcal{I}_i})^+, \text{ if } \mathsf{Trans}_i(R) \in \mathcal{R}_i \\
(R^-)^{\mathcal{I}_i} &= \{(x, y) | (y, x) \in R^{\mathcal{I}_i}\} \\
(C \sqcap D)^{\mathcal{I}_i} &= C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i} \\
(C \sqcup D)^{\mathcal{I}_i} &= C^{\mathcal{I}_i} \cup D^{\mathcal{I}_i} \\
(\neg_j C)^{\mathcal{I}_i} &= r_{ji}(\Delta^{\mathcal{I}_j}) \backslash C^{\mathcal{I}_i} \\
(\exists R.C)^{\mathcal{I}_i} &= \{x \in r_{ji}(\Delta^{\mathcal{I}_j}) | \exists y \in \Delta^{\mathcal{I}_i}, (x, y) \in R^{\mathcal{I}_i} \wedge y \in C^{\mathcal{I}_i}\} \\
(\forall R.C)^{\mathcal{I}_i} &= \{x \in r_{ji}(\Delta^{\mathcal{I}_j}) | \forall y \in \Delta^{\mathcal{I}_i}, (x, y) \in R^{\mathcal{I}_i} \rightarrow y \in C^{\mathcal{I}_i}\} \\
(\geqslant nS.C)^{\mathcal{I}_i} &= \{x \in r_{ji}(\Delta^{\mathcal{I}_j}) | \, |\{y \in \Delta^{\mathcal{I}_i} | (x, y) \in S^{\mathcal{I}_i} \wedge y \in C^{\mathcal{I}_i}\}| \geqslant n\} \\
(\leqslant nS.C)^{\mathcal{I}_i} &= \{x \in r_{ji}(\Delta^{\mathcal{I}_j}) | \, |\{y \in \Delta^{\mathcal{I}_i} | (x, y) \in S^{\mathcal{I}_i} \wedge y \in C^{\mathcal{I}_i}\}| \leqslant n\}
\end{aligned}
$$

Note that, when $i = j$, since $r_{ii} = \mathrm{id}_{\Delta^{\mathcal{I}_i}}$, $(\neg_j C)^{\mathcal{I}_i}$ reduces to the usual negation $(\neg_i C)^{\mathcal{I}_i} = \Delta^{\mathcal{I}_i} \backslash C^{\mathcal{I}_i}$. Similarly, the other semantic definitions also reduce to the usual DL semantic definitions.

For an example of contextualized negation, suppose $A = C^{\mathcal{I}_i}$ in the Figure 13.2, then $(\neg_i C)^{\mathcal{I}_j}$ will only contain y_2 but not y_3. On the other hand, $(\neg_j C)^{\mathcal{I}_j}$ is will contain both y_2 and y_3.

A local interpretation $\mathcal{I}_i$ *satisfies* a role inclusion axiom $R_1 \sqsubseteq R_2$ iff $R_1^{\mathcal{I}_i} \subseteq R_2^{\mathcal{I}_i}$ and a GCI $C \sqsubseteq D$ iff $C^{\mathcal{I}_i} \subseteq D^{\mathcal{I}_i}$. $\mathcal{I}_i$ is a *model* of P_i, denoted by $\mathcal{I}_i \models P_i$, if it satisfies all axioms in P_i.

The proposed semantics of $\mathcal{SHOIQP}$ is motivated by the need to overcome some of the limitations of existing approaches that can be traced back to the arbitrary construction of domain relations and the lack of support for contextualized interpretation. Specifically, we seek a semantics that satisfies the following desiderata:

- **Preservation of concept unsatisfiability.** The intuition is that an unsatisfiable concept expression should never be reused so as to be interpreted as a satisfiable concept. Formally, we say that a domain relation r_{ij} *preserves the unsatisfiability* of a concept C, that appears in both P_i and P_j, if whenever $C^{\mathcal{I}_i} = \emptyset$, it is necessarily the case that $C^{\mathcal{I}_j} = \emptyset$.
- **Transitive reusability of knowledge.** The intention is that the consequences of some of the axioms in one module can be propagated in a transitive fashion to other ontology modules. For example, if a package P_i asserts that $C \sqsubseteq D$, and P_j directly or indirectly imports that axiom from P_i, then it should be the case that $C \sqsubseteq D$ is also valid from the *point of view* of P_j.
- **Contextualized interpretation of knowledge.** The idea is that the interpretation of assertions in each ontology module is constrained by their context. When knowledge, e.g., axioms, in that module is reused by other modules, the interpretation of the reused knowledge should be constrained by the context in which the knowledge is being reused.
- **Improved expressivity.** Ideally, the language should support
  1. both inter-module concept inclusion and concept construction using foreign concepts, roles and nominals;
  2. more general reuse of roles and of nominals than allowed by existing approaches.

A major goal of this chapter is to explore the constraints that need to be imposed on local interpretations so that the resulting semantics for $\mathcal{SHOIQP}$ satisfies the desiderata enumerated above. These constraints are presented in the following:

Definition 1. *An interpretation* $\mathcal{I} = \langle \{\mathcal{I}_i\}, \{r_{ij}\}_{P_i \in P_j^*} \rangle$ *is a* model *of a* $\mathcal{SHOIQP}$ *KB* $\Sigma = \{P_i\}$, *denoted as* $\mathcal{I} \models \Sigma$, *if* $\bigcup_i \Delta^{\mathcal{I}_i} \neq \emptyset$, *i.e., at least one of the local interpretation domains is non-empty*[4], *and the following conditions are satisfied:*

[4] This agrees with conventional model-theoretic semantics, where an ordinary model (of a single package) is assumed to have a non-empty domain.

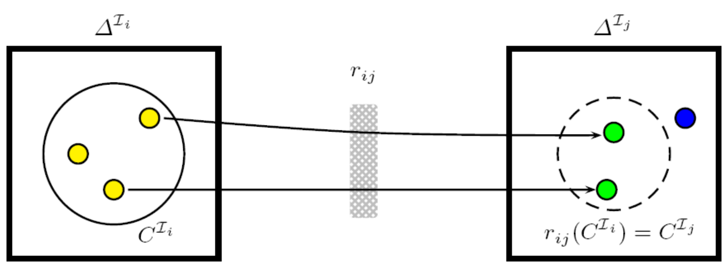
1. *For all* i, j, r_{ij} *is one-to-one, i.e., it is an injective partial function.*
2. Compositional Consistency*: For all* i, j, k *s.t.* $P_i \in P_k^*$ *and* $P_k \in P_j^*$*, we have* $\rho_{ij} = r_{ij} = r_{kj} \circ r_{ik}$.
3. *For every* i*-concept name* C *that appears in* P_j*, we have* $r_{ij}(C^{\mathcal{I}_i}) = C^{\mathcal{I}_j}$.
4. *For every* i*-role* R *that appears in* P_j*, we have* $R^{\mathcal{I}_j} = r_{ij}(R^{\mathcal{I}_i})$.
5. Cardinality Preservation for Roles*: For every* i*-role* R *that appears in* P_j *and every* $(x, x') \in r_{ij}$, $y \in R^{\mathcal{I}_i}(x)$ *iff* $r_{ij}(y) \in R^{\mathcal{I}_j}(x')$.
6. *For every* i*-nominal* o *that appears in* P_j, $(o^{\mathcal{I}_i}, o^{\mathcal{I}_j}) \in r_{ij}$.
7. $\mathcal{I}_i \models P_i$*, for every* i.

The proposed semantics for $\mathcal{SHOIQP}$ is an extension of the semantics for $\mathcal{ALCP_C}$ [4], which uses Conditions 1,2,3 and 7 above, and borrows Condition 5 from the semantics of Semantic Importing [20].

Intuitively, one-to-oneness (Condition 1, see Figure 13.4) and compositional consistency (Condition 2, Figure 13.5) ensure that the parts of local domains connected by domain relations match perfectly. Conditions 3 and 4 ensure consistency between the interpretations of concepts and of roles in their home package and the interpretations in the packages that import them. Condition 5 (Figure 13.6) ensures that r_{ij} is a total bijection from $R^{\mathcal{I}_i}(x)$ to $R^{\mathcal{I}_j}(r_{ij}(x))$. In particular, the sizes $|R^{\mathcal{I}_i}(x)|$ and $|R^{\mathcal{I}_j}(r_{ij}(x))|$ are always equal in different local domains. Condition 6 ensures the uniqueness of nominals. In Section 4, we will show that Conditions 1-7 are minimally sufficient to guarantee that the desiderata for the semantics of $\mathcal{SHOIQP}$ as outlined above are indeed satisfied.

Note that Condition 2 implies that if P_i and P_j mutually (possibly indirectly) import one another, then $r_{ij} = \rho_{ij} = \rho_{ji}^- = r_{ji}^-$ and r_{ij} is a total function from $\Delta^{\mathcal{I}_i}$ to $\Delta^{\mathcal{I}_j}$. However, if $P_j \notin P_i^*$, r_{ji} does not necessarily exist even if r_{ij} exists. In that case, r_{ij} is not necessarily a total function.

Definition 2. *An ontology* Σ *is* consistent as witnessed by a package P_w *of* Σ *if* P_w^* *has a model* $\mathcal{I} = \langle \{\mathcal{I}_i\}, \{r_{ij}\}_{P_i \in P_j^+} \rangle$*, such that* $\Delta^{\mathcal{I}_w} \neq \emptyset$*. A concept* C *is* satisfiable as witnessed by P_w *if there is a model* $\mathcal{I}$ *of* P_w^**, such that* $C^{\mathcal{I}_w} \neq \emptyset$*. A concept*



An image domain relation in P-DL is one-to-one, i.e., it is a partial injective function. It is not necessarily total, i.e., some individuals of $C^{\mathcal{I}_i}$ may not be mapped to $\Delta^{\mathcal{I}_j}$.

Fig. 13.4. One-to-One Domain Relation

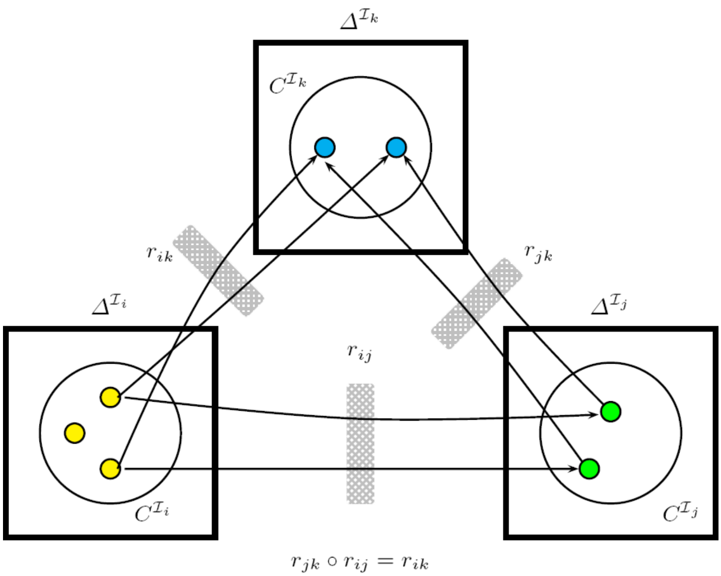
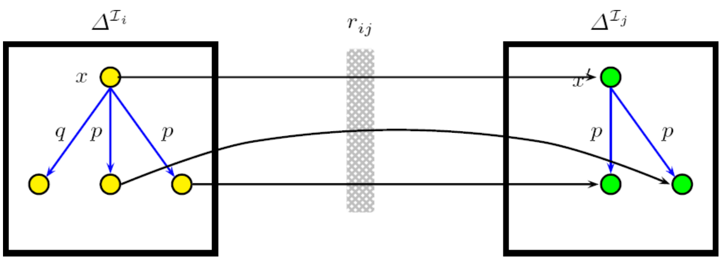


Fig. 13.5. Compositionally Consistent Domain Relation



If an i-role p is imported by P_j, then every pair of p instances must have a "preimage" pair in Δ_i. The cardinality preservation condition for roles, illustrated in this figure, requires that, if an individual x in $\Delta^{\mathcal{I}_i}$ has an image individual x' in $\Delta^{\mathcal{I}_j}$, then each of its p-neighbors must have an image in $\Delta^{\mathcal{I}_j}$ which is a p-neighbor of x'.

Fig. 13.6. Cardinality Preservation for Roles

subsumption $C \sqsubseteq D$ is valid as witnessed by P_w, *denoted by* $C \sqsubseteq_w D$, *if, for every model* $\mathcal{I}$ *of* P_w^*, $C^{\mathcal{I}_w} \subseteq D^{\mathcal{I}_w}$.

Hence, in $\mathcal{SHOIQP}$, the questions of consistency, satisfiability and subsumption are always answered from the local point of view of a *witness package* and it is possible that different packages draw different conclusions from their own points of view.

The following examples show some inference problems that a P-DL ontology can tackle. Precise proofs for general cases will be given in Section 13.4.

Example 4. *Transitive subsumption propagation. Given three packages:* $P_1 : \{1 : A \sqsubseteq 1 : B\}$, $P_2 : \{1 : B \sqsubseteq 2 : C\}$, $P_3 : \{2 : C \sqsubseteq 3 : D\}$, *the subsumption query* $1 : A \sqsubseteq 3 : D$ *is answered in the affirmative as witnessed by* P_3.

Example 5. *Detection of inter-module unsatisfiability. Given two packages* $P_1 : \{1 : B \sqsubseteq 1 : F\}$, $P_2 : \{1 : P \sqsubseteq 1 : B, 2 : P \sqsubseteq \neg 1 : F\}$, $2 : P$ *is unsatisfiable as witnessed by* P_2.

Example 6. *Reasoning from a local point of view. Given two packages* $P_1 : \{1 : A \sqsubseteq 1 : C\}$, $P_2 : \{1 : A \sqsubseteq \exists 2 : R.(2 : B), 2 : B \sqsubseteq 1 : A \sqcap (\neg 1 : C)\}$, *consider the satisfiability of* $1 : A$ *as witnessed by* P_1 *and* P_2*, respectively. It is easy to see* A *is satisfiable when witnessed by* P_1*, but unsatisfiable when witnessed by* P_2*. Thus, inferences in P-DL are always drawn from the point of view of a witness package. Different witnesses, because they operate on different domains, and have access to different pieces of knowledge, can draw dramatically different conclusions.*

Discussion: Relation between the Semantics of P-DL and Partially-Overlapping Local Domain Semantics

In [10] a semantics based on partially overlapping domains was proposed for terminology mappings between ontology modules. In that framework, a global interpretation $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ is given together with local domains $\Delta^{\mathcal{I}_i}$, that are subsets of $\Delta^{\mathcal{I}}$. Any two local domains may be partially overlapping. Moreover, inclusions between concepts are of the following two forms:

- $i : C \sqsubseteq_{\text{ext}} j : D$ (extensional inclusion), with semantics $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$, and
- $i : C \sqsubseteq_{\text{int}} j : D$ (intentional inclusion), with semantics $C^{\mathcal{I}} \cap \Delta^{\mathcal{I}_i} \cap \Delta^{\mathcal{I}_j} \subseteq D^{\mathcal{I}} \cap \Delta^{\mathcal{I}_i} \cap \Delta^{\mathcal{I}_j}$.

Since P-DL semantics does not envision a global point of view, extensional inclusion has no corresponding notion in P-DL semantics. In addition, P-DL semantics differs significantly from this approach in that, while both intentional and extensional inclusions are not directional, the semantic importing in P-DL is. To make this distinction clearer, consider two packages P_i and P_j, such that $P_i \mapsto P_j$. Let C, D be two i-concept names that are imported by P_j and consider the interpretation where $\Delta^{\mathcal{I}_i} = \{x, y, z\}, \Delta^{\mathcal{I}_j} = \{y, z\}$, $C^{\mathcal{I}_i} = \{x, y\}, D^{\mathcal{I}_i} = \{y, z\}$ and $r_{ij} = \{\langle y, y\rangle, \langle z, z\rangle\}$. Then, in P-DL, from the point of view of package P_i, we have $C^{\mathcal{I}_i} = \{x, y\} \not\subseteq \{y, z\} = D^{\mathcal{I}_i}$. Therefore, $\mathcal{I} \not\models_i C \sqsubseteq D$. Similarly, from the point of view of package P_j, we have $C^{\mathcal{I}_j} = r_{ij}(C^{\mathcal{I}_i}) = r_{ij}(\{x, y\}) = \{y\} \subseteq \{y, z\} = r_{ij}(\{y, z\}) = r_{ij}(D^{\mathcal{I}_i}) = D^{\mathcal{I}_j}$. Therefore, $\mathcal{I} \models_j C \sqsubseteq D$. However, in the partially overlapping domain semantics of [10], $C =_{\text{int}} D$ holds from both P_i's and P_j's point of view.

Thus, in spite of the fact that the intersection of two sets is “seen equally” from both sets’ points of view, the example that was presented above illustrates that the way concept names are interpreted in these models still preserves some form of directionality in the subsumption reasoning.

Despite this subtle semantic difference between the partially overlapping domain semantics of [10] and the semantics of P-DL presented here, it is still possible to provide P-DL with a different kind of overlapping-domain-style semantics. More precisely, in the proof of Lemma 3, it is shown how one may combine the various local domains of a P-DL interpretation into one global domain. The P-DL model satisfies a given subsumption $C \sqsubseteq D$ from a witness P_i's point of view if and only if the global model satisfies an appropriately constructed *subjective* translation $\#_i(C) \sqsubseteq \#_i(D)$ of the given subsumption (see Section 3). Moreover, in the proof of Lemma 2, it is shown how, conversely, starting from a global domain, one may construct a P-DL model with various local domains; if the aforementioned subjective translation of a subsumption is satisfied in the global domain, then the original subsumption is satisfied from P_i's point of view. If the two constructions are composed, starting from the original P-DL model one obtains another equivalent model that is based on a partially-overlapping-style domain semantics. However, due to the interpretations of the translations of the concept names in this model, directionality is still preserved, unlike the situation in the ordinary partially overlapping domain semantics of [10].

Since any ordinary P-DL model gives rise to an equivalent model with partially-overlapping-style semantics, the question arises as to why the latter is not chosen as the fundamental notion of semantics for P-DL. The main reason is that, in many applications, local models are supposed to be populated independently of one another before semantic relations between their individuals are physically established. Moreover, the whole point of introducing modular description logics is to give temporally and spatially unrelated designers the chance to develop modules of a complex knowledge base independently. Additionally, the semantics of P-DL is derived from the Local Model Semantics [11], of which the directionality of domain relations, which will be lost in the partially-overlapping-style semantics, are crucial as domain relations also subjective. By keeping the directionality of domain relations, it also opens the possibility for various future extensions of P-DL when it is infeasible to use partially-overlapping-style semantics, e.g., when transitive knowledge propagation should be controlled among only trusted entities. □ (**End of Discussion**)

As immediate consequences of the proposed semantics for the P-DL $\mathcal{SHOIQP}$, extensions of various versions of the De Morgan's Law may be proven. Those deal with both the ordinary propositional logical connectives, including local negations, and with the quantifiers. For instance, it may be shown that, from the point of view of a package P_j which directly imports packages P_i and P_k, we have that $\neg_i(C \sqcap D) = \neg_i C \sqcup \neg_i D$ and also, $\neg_i(\forall R.C) = \neg_i \top_k \sqcup \exists R.\neg_j C$, where R is a k-role name. Similar semantic equivalences hold for various other connectives and quantifiers. Via these relations, proofs involving existential restriction and value restriction may be reduced to those involving the corresponding number restrictions.

In the next lemma, it is asserted that Condition 3 of Definition 1 holds not only for concept names, but, in fact, for arbitrary concepts. Beyond its own intrinsic interest, it becomes handy in Section 4 in showing that the package description logic $\mathcal{SHOIQP}$ supports monotonicity of reasoning and transitive reusability of modules.

Lemma 1. *Let Σ be a $\mathcal{SHOIQP}$ ontology, P_i, P_j two packages in Σ such that $P_i \in P_j^+$, C a concept such that* $\mathsf{Sig}(C) \subseteq \mathsf{Sig}(P_i) \cap \mathsf{Sig}(P_j)$*, and R a role name such that $R \in \mathsf{Sig}(P_i) \cap \mathsf{Sig}(P_j)$. If $\mathcal{I} = \langle \{\mathcal{I}_u\}, \{r_{uv}\}_{P_u \in P_v^+} \rangle$ is a model of Σ, then $r_{ij}(C^{\mathcal{I}_i}) = C^{\mathcal{I}_j}$ and $r_{ij}(R^{\mathcal{I}_i}) = R^{\mathcal{I}_j}$.*

The proof of Lemma 1 involves a structural induction on the concept formula C, that, by hypothesis, appears both in P_i and in P_j. The induction step employs the fact that, if $x' = r_{ij}(x)$, then

- $r_{ij} : R^{\mathcal{I}_i}(x) \rightarrow R^{\mathcal{I}_j}(x')$ is a total bijection and
- $r_{ij} : R^{\mathcal{I}_i}(x) \cap D^{\mathcal{I}_i} \rightarrow R^{\mathcal{I}_j}(x') \cap D^{\mathcal{I}_j}$ is also a total bijection, for every concept D, that appears in both P_i and P_j, and is such that $r_{ij}(D^{\mathcal{I}_i}) = D^{\mathcal{I}_j}$.

13.3 Reduction to Ordinary DL

In this section, we present a translation from concept formulas that appear in a given package of a $\mathcal{SHOIQP}$ KB Σ to concept formulas of a $\mathcal{SHOIQ}$ KB $\Sigma^\star$. The $\mathcal{SHOIQ}$ KB $\Sigma^\star$ is constructed in such a way that the top concept $\top_w$, associated with a specific package P_w of Σ, is satisfiable by $\Sigma^\star$ in the ordinary DL sense if and only if Σ itself is consistent *from the point of view of* P_w (see Theorem 1). (Note that the $\mathcal{SHOIQ}$ KB $\Sigma^\star$ is dependent on the importing relations present in $\mathcal{SHOIQP}$ Σ). This shows that the consistency problem in $\mathcal{SHOIQP}$ is reducible to the satisfiability problem in $\mathcal{SHOIQ}$, which is known to be NExpTime-complete [23, 24]. This has the consequence that the problems of concept satisfiability, concept subsumption and consistency in $\mathcal{SHOIQP}$ are also NExpTime-complete (see Theorem 2). Moreover, as will be seen in Section 4, this result also plays a central role in showing that some of the desiderata presented in Section 2.2 are satisfied by $\mathcal{SHOIQP}$. For instance, Reasoning Exactness, Monotonicity of Reasoning, Transitive Reusability of Knowledge and Preservation of Unsatisfiability are all features of $\mathcal{SHOIQP}$, which are shown to hold by employing the translation from $\mathcal{SHOIQP}$ to $\mathcal{SHOIQ}$, that will be presented in this section.

The reduction $\Re$ from a $\mathcal{SHOIQP}$ KB $\Sigma = \{P_i\}$ to a $\mathcal{SHOIQ}$ KB $\Sigma^\star$ can be obtained as follows: the signature of $\Sigma^\star$ is the union of the local signatures of the component packages together with a global top $\top$, a global bottom $\bot$ and local top concepts $\top_i$, for all i, i.e., $\mathsf{Sig}(\Sigma^\star) = \bigcup_i (\mathsf{Loc}(P_i) \cup \{\top_i\}) \cup \{\top, \bot\}$, and

a) For all i, j, k such that $P_i \in P_k^*$, $P_k \in P_j^*$, $\top_i \sqcap \top_j \sqsubseteq \top_k$ is added to $\Sigma^\star$.
b) For each GCI $X \sqsubseteq Y$ in P_j, $\#_j(X) \sqsubseteq \#_j(Y)$ is added to Σ. The mapping $\#_j()$ is defined below.
c) For each role inclusion $X \sqsubseteq Y$ in P_j, $X \sqsubseteq Y$ is added to $\Sigma^\star$.
d) For each i-concept name or i-nominal name C in P_i, $i : C \sqsubseteq \top_i$ is added to $\Sigma^\star$.
e) For each i-role name R in P_i, $\top_i$ is stipulated to be its domain and range, i.e., $\top \sqsubseteq \forall R^-.\top_i$ and $\top \sqsubseteq \forall R.\top_i$ are added to Σ.

f) For each i-role name R in P_j, the following axioms are added to $\Sigma^\star$:
   – $\exists R.\top_j \sqsubseteq \top_j$ (local domain);
   – $\exists R^-.\top_j \sqsubseteq \top_j$ (local range).

g) For each i-role name, add $\mathsf{Trans}(R)$ to $\Sigma^\star$ if $\mathsf{Trans}_i(R)$.

The mapping $\#_j()$ is adapted from a similar one for DDL [8] with modifications to facilitate context preservation whenever name importing occurs. For a formula X used in P_j, $\#_j(X)$ is:

- X, for a j-concept name or a j-nominal name.
- $X \sqcap \top_j$, for an i-concept name or an i-nominal name X.
- $\neg\#_j(Y) \sqcap \top_i \sqcap \top_j$, for $X = \neg_i Y$, where Y is a concept.
- $(\#_j(X_1) \oplus \#_j(X_2)) \sqcap \top_j$, for a concept $X = X_1 \oplus X_2$, where $\oplus = \sqcap$ or $\oplus = \sqcup$.
- $(\otimes R.\#_j(X')) \sqcap \top_i \sqcap \top_j$, for a concept $X = (\otimes R.X')$, where $\otimes \in \{\exists, \forall, \leq n, \geq n\}$ and R is an i-role.

For example, if C, D are concept names and R a role name,

$$\begin{aligned}
\#_j(\neg_i\, i:C) &= \neg(C \sqcap \top_j) \sqcap \top_i \sqcap \top_j \\
\#_j(j:D \sqcup i:C) &= (D \sqcup (C \sqcap \top_j)) \sqcap \top_j \\
\#_j(\forall(j:R).(i:C)) &= \forall R.(C \sqcap \top_j) \sqcap \top_j \\
\#_j(\exists(i:R).(i:C)) &= \exists R.(C \sqcap \top_j) \sqcap \top_i \sqcap \top_j
\end{aligned}$$

It should be noted that $\#_j()$ is *contextualized* so as to allow a given formula to have different interpretations when it appears in different packages. See also the Discussion subsection in Section 2.2.

13.4 Properties of Semantic Importing

In this section, we further justify the proposed semantics for $\mathcal{SHOIQP}$. More specifically, we present the main results showing that $\mathcal{SHOIQP}$ satisfies the desiderata listed in Section 2.

The first main theorem shows that the consistency problem of a $\mathcal{SHOIQP}$ ontology w.r.t. a witness package can be reduced to a satisfiability problem of a $\mathcal{SHOIQ}$ concept w.r.t. an integrated ontology *from the point of view of that witness package*, namely, $\Re(P_w^*)$. Note that there is no single universal integrated ontology for all packages. Each package, *sees* an integrated ontology (depending on the witness package and all the packages that are directly or indirectly imported by the witness package), and hence different packages can witness different consequences.

Theorem 1. *A $\mathcal{SHOIQP}$ KB Σ is consistent as witnessed by a package P_w if and only if $\top_w$ is satisfiable with respect to $\Re(P_w^*)$.*

Proof: Sufficiency is proven in Lemma 2 and necessity in Lemma 3. We present these two lemmas below, but give only outlines of their proofs. Detailed proofs are provided in [6].

Lemma 2. *Let Σ be a $\mathcal{SHOIQP}$ KB and P_w a package of Σ. If $\top_w$ is satisfiable with respect to $\Re(P_w^*)$, then Σ is consistent as witnessed by P_w.*

Proof: Assume that $\top_w$ is satisfiable with respect to $\Re(P_w^*)$ and let $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ be a model of $\Re(P_w^*)$, such that $\top_w^{\mathcal{I}} \neq \emptyset$. We construct a model $\langle \{\mathcal{I}_i\}, \{r_{ij}\}_{i \in P_j^*} \rangle$ of P_w^*, such that $\Delta^{\mathcal{I}_w} \neq \emptyset$. For each package $P_i \in P_w^*$, the local interpretation $\mathcal{I}_i$ is constructed as a projection of $\mathcal{I}$ in the following way:

- $\Delta^{\mathcal{I}_i} = \top_i^{\mathcal{I}}$;
- For every concept name C that appears in P_i, $C^{\mathcal{I}_i} = C^{\mathcal{I}} \cap \top_i^{\mathcal{I}}$;
- For every role name R that appears in P_i, $R^{\mathcal{I}_i} = R^{\mathcal{I}} \cap (\top_i^{\mathcal{I}} \times \top_i^{\mathcal{I}})$;
- For every nominal name o that appears in P_i, $o^{\mathcal{I}_i} = o^{\mathcal{I}}$;

and for every pair i, j, such that $P_i \in P_j^* \subseteq P_w^*$, we define

$$r_{ij} = \{(x, x) | x \in \Delta^{\mathcal{I}_i} \cap \Delta^{\mathcal{I}_j}\}.$$

Clearly, we have $\Delta^{\mathcal{I}_w} = \top_w^{\mathcal{I}} \neq \emptyset$, by the hypothesis. Moreover, it may be shown that $\langle \{\mathcal{I}_i\}, \{r_{ij}\}_{P_i \in P_j^*} \rangle$ is a model of the modular ontology P_w^*, i.e., that it satisfies the seven conditions postulated in Definition 1. The most challenging part is to show that, for every concept inclusion $C \sqsubseteq D$ in P_j, we must have $C^{\mathcal{I}_j} \subseteq D^{\mathcal{I}_j}$. Since, by the hypothesis, $\#_j(C)^{\mathcal{I}} \subseteq \#_j(D)^{\mathcal{I}}$ holds in $\mathcal{I}$, it suffices to show that, for every concept formula X that appears in P_j, we have $\#_j(X)^{\mathcal{I}} = X^{\mathcal{I}_j}$. This may be accomplished by structural induction on X. The details are omitted. Q.E.D.

Next, we proceed to show the reverse implication.

Lemma 3. *Let Σ be a $\mathcal{SHOIQP}$ KB. If Σ is consistent as witnessed by a package P_w, then $\top_w$ is satisfiable with respect to $\Re(P_w^*)$.*

Proof: Suppose that Σ is consistent as witnessed by P_w. Thus, it has a distributed model $\langle \{\mathcal{I}_i\}, \{r_{ij}\}_{P_i \in P_j^*} \rangle$, such that $\Delta^{\mathcal{I}_w} \neq \emptyset$. We proceed to construct a model $\mathcal{I}$ of $\Re(P_w^*)$ by merging individuals that are related via chains of image domain relations or their inverses. More precisely, for every element x in the distributed model, we define its equivalence class $\overline{x} = \{y | (x, y) \in \rho\}$ where ρ is the symmetric and transitive closure of the set $\bigcup_{P_i \in P_j^*} r_{ij}$. Moreover, for a set S, we define $\overline{S} = \{\bar{x} | x \in S\}$ and for a binary relation R, we define $\overline{R} = \{(\overline{x}, \overline{y}) | (x, y) \in R\}$.

A model $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ of Σ is now defined as follows:

- $\top^{\mathcal{I}} = \Delta^{\mathcal{I}} = \overline{\bigcup_i \Delta^{\mathcal{I}_i}}$, and $\bot^{\mathcal{I}} = \emptyset$.
- For every i-name X, $X^{\mathcal{I}} := \overline{X^{\mathcal{I}_i}}$.
- For every i, $\top_i^{\mathcal{I}} = \overline{\Delta^{\mathcal{I}_i}}$.

Next, it is shown that $\mathcal{I}$ is a model of $\Re(P_w^*)$, such that $\top_w^{\mathcal{I}} \neq \emptyset$. As in the proof of Lemma 2, the most challenging part is to show that, if $C \sqsubseteq D$ appears in P_j, then $\#_j(C)^{\mathcal{I}} \subseteq \#_j(D)^{\mathcal{I}}$ holds in $\mathcal{I}$. Since, by hypothesis, $C^{\mathcal{I}_j} \subseteq D^{\mathcal{I}_j}$ and this implies that $\overline{C^{\mathcal{I}_j}} \subseteq \overline{D^{\mathcal{I}_j}}$, it suffices to show that $\#_j(C)^{\mathcal{I}} = \overline{C^{\mathcal{I}_j}}$, for every concept formula C that appears in P_j. This is accomplished by induction on the structure of the concept C. The details can be found in [6]. Q.E.D.

Using Theorem 1 and the fact that concept satisfiability in $\mathcal{SHOIQ}$ is NExpTime-complete [23, 24], we obtain

Theorem 2. *The concept satisfiability, concept subsumption and consistency problems in $\mathcal{SHOIQP}$ are* NExpTime*-complete.*

The next theorem shows that concept subsumption problems in a $\mathcal{SHOIQP}$ ontology Σ, from the point of view of a specific witness package, can be reduced to concept subsumption problems in a *corresponding* $\mathcal{SHOIQ}$ ontology.

Theorem 3 (Reasoning Exactness). *For a $\mathcal{SHOIQP}$ KB $\Sigma = \{P_i\}$, $C \sqsubseteq_j D$ iff $\Re(P_j^*) \models \#_j(C) \sqsubseteq \#_j(D)$.*

Proof: As usual, we reduce subsumption to (un)satisfiability. It follows directly from Theorem 1 that P_j^* and $C \sqcap \neg_j D$ have a common model if and only if $\Re(P_j^*)$ and $\#_j(C) \sqcap \neg\#_j(D) \sqcap \top_j$ have a common model. Since $\#_j(C) \sqsubseteq \top_j$, this holds if and only if $\Re(P_j^*)$ and $\#_j(C) \sqcap \neg\#_j(D)$ have a common model. Thus, $\Re(P_j^*) \models \#_j(C) \sqsubseteq \#_j(D)$. Q.E.D.

Discussion of Desiderata

To show that the package description logic $\mathcal{SHOIQP}$ supports transitive reusability and preservation of unsatisfiability, we prove the monotonicity of reasoning in $\mathcal{SHOIQP}$.

Theorem 4 (Monotonicity and Transitive Reusability). *Suppose $\Sigma = \{P_i\}$ is a $\mathcal{SHOIQP}$ KB, $P_i \in P_j^+$ and C, D are concepts, such that $\mathsf{Sig}(C) \cup \mathsf{Sig}(D) \subseteq \mathsf{Sig}(P_i) \cap \mathsf{Sig}(P_j)$. If $C \sqsubseteq_i D$, then $C \sqsubseteq_j D$.*

Proof: Suppose that $C \sqsubseteq_i D$. Thus, for every model $\mathcal{I}$ of P_i^*, $C^{\mathcal{I}_i} \subseteq D^{\mathcal{I}_i}$. Now consider a model $\mathcal{J}$ of P_j^*. Since $P_i \in P_j^*$, $\mathcal{J}$ is also an interpretation of P_i^*. If $\bigcup_{P_k \in P_i^*} \Delta^{\mathcal{J}_k} = \emptyset$, then the conclusion holds trivially. Otherwise, $\mathcal{J}$ is a model of P_i^* and, therefore, $C^{\mathcal{J}_i} \subseteq D^{\mathcal{J}_i}$. Hence, $r_{ij}(C^{\mathcal{J}_i}) \subseteq r_{ij}(D^{\mathcal{J}_i})$, whence, by Lemma 1, $C^{\mathcal{J}_j} \subseteq D^{\mathcal{J}_j}$. This proves that $C \sqsubseteq_j D$. Q.E.D.

Theorem 4 ensures that when some part of an ontology module is reused, the restrictions asserted by it, e.g., domain restrictions on roles, will not be relaxed in a way that prohibits the reuse of imported knowledge. Theorem 4 also ensures that consequences of imported knowledge can be transitively propagated across importing chains.

In the special case where $D = \bot$, we obtain the following corollary:

Corollary 1 (Preservation of Unsatisfiability). *For a $\mathcal{SHOIQP}$ knowledge base $\Sigma = \{P_i\}$ and $P_i \in P_j^+$, if $C \sqsubseteq_i \bot$ then $C \sqsubseteq_j \bot$.*

Finally, the semantics of $\mathcal{SHOIQP}$ ensures that the interpretation of an axiom in an ontology module is constrained by its *context*, as seen from the reduction to a corresponding integrated ontology: $C \sqsubseteq D$ in P_j is mapped to $\#_j(C) \sqsubseteq \#_j(D)$, where $\#_j(C)$ and $\#_j(D)$ are now relativized to the corresponding local domain of P_j.

When a package P_i is directly or indirectly reused by another package P_j, some axioms in P_i may be effectively "propagated" to module P_j (i.e., may influence inference from the point of view of P_j). P-DL semantics ensures that such axiom propagation will affect only the "overlapping" domain $r_{ij}(\Delta^{\mathcal{I}_i}) \cap \Delta^{\mathcal{I}_j}$ and not the entire domain $\Delta^{\mathcal{I}_j}$.

Example 7. *For instance, in Figure 13.1, package* P_1 *contains an axiom* $\neg_1\mathsf{Child} \sqsubseteq \mathsf{Adult}$ *and package* P_2 *imports* P_1. *The assertion* $\neg_1\mathsf{Child} \sqsubseteq \mathsf{Adult}$ *is made within the implicit context of people, i.e. every individual that is not a child is an adult. Thus, every individual within the domain of people are either a* Child *or an* Adult *(*$\top_1 \sqsubseteq \mathsf{Child} \sqcup \mathsf{Adult}$*). However, it is not necessarily the case in* P_2 *that* $\top_2 \sqsubseteq \mathsf{Child} \sqcup \mathsf{Adult}$. *For example, an* Empolyer *in the domain of Work may be an organization which is not a member of the domain of People. In fact, since* $r_{12}(\Delta^{\mathcal{I}_1}) \subseteq \Delta^{\mathcal{I}_2}$, $\Delta^{\mathcal{I}_1} \backslash \mathsf{Child}^{\mathcal{I}_1} \subseteq \mathsf{Adult}^{\mathcal{I}_1}$, *i.e.,* $\Delta^{\mathcal{I}_1} = \mathsf{Child}^{\mathcal{I}_1} \cup \mathsf{Adult}^{\mathcal{I}_1}$, *does not necessarily imply* $\Delta^{\mathcal{I}_2} = \mathsf{Child}^{\mathcal{I}_2} \cup \mathsf{Adult}^{\mathcal{I}_2}$.

Hence, the effect of an axiom is always limited to its original designated context. Consequently, it is not necessary to explicitly restrict the use of the ontology language to ensure locality of axioms, as is required, for instance, by conservative extensions [13]. Instead, the locality of axioms follows directly from the semantics of $\mathcal{SHOIQP}$.

13.5 Discussion of the P-DL Semantics

13.5.1 Necessity of P-DL Constraints on Domain Relations

The constraints on domain relations in the semantics of $\mathcal{SHOIQP}$, as given in Definition 1, are minimal in the sense that if we drop any of them, we can no longer satisfy the desiderata summarized in Section 13.2.2.

Dropping Condition 1 of Definition 1 (one-to-one domain relations) leads to difficulties in preservation of concept unsatisfiability. For example, if the domain relations are not injective, then $C_1 \sqsubseteq_i \neg_i C_2$, i.e., $C_1 \sqcap C_2 \sqsubseteq_i \bot$, does not ensure $C_1 \sqcap C_2 \sqsubseteq_j \bot$ when P_j imports P_i. If the domain relations are not partial functions, multiple individuals in $\Delta^{\mathcal{I}_j}$ may be images of the same individual in $\Delta^{\mathcal{I}_i}$ via r_{ij}, whence unsatisfiability of a complex concept can no longer be preserved when both number restriction and role importing are allowed. Thus, if R is an i-role name and C is an i-concept name, $\geq 2R.C \sqsubseteq_i \bot$ does not imply $\geq 2R.C \sqsubseteq_j \bot$.

Dropping Condition 2 of Definition 1 (compositional consistency of domain relations) would result in violation of the transitive reusability requirement, in particular, and of the monotonicity of inference based on imported knowledge, in general. In the absence of compositional consistency of domain relations, the importing relations would be like bridge rules in DDL, in that they are localized w.r.t. the connected pairs of modules without supporting compositionality [25].

In the absence of Conditions 3 and 4 of Definition 1, the reuse of concept and role names would be purely syntactical, i.e., the local interpretations of imported concepts and role names would be unconstrained by their interpretations in their home package.

Condition 5 (cardinality preservation of role instances) is needed to ensure the consistency of local interpretations of complex concepts that use number restrictions.

Condition 6 is needed to ensure that concepts that are nominals can only have one instance. Multiple "copies" of such an instance are effectively identified with a single instance via domain relations.

Finally, Condition 7, i.e., that $\mathcal{I}_i \models P_i$, for every i, is self-explanatory.

13.5.2 Contextualized Negation

Contextualized negation has been studied in logic programming [21, 22]. Existing modular ontology languages DDL and $\mathcal{E}$-Connections do not explicitly support contextualized negation in their respective syntax. In fact, in those formalisms, a negation is always interpreted with respect to the local domain of the module in which the negation occurs, not the union of all local domains. Thus, in fact, both DDL and $\mathcal{E}$-Connections implicitly support contextualized negation.

The P-DL syntax and semantics, proposed in this work, support a more general use of contextualized negation so that a package can use, besides its own negation, the negations of its imported packages[5].

13.5.3 Directionality of Importing

There appears to be some apparent confusion in the literature regarding whether the constraints imposed by P-DL allow the importing relations in P-DL to be indeed directional [15]. As noted by Grau [15], if it is indeed the case that a P-DL model $\mathcal{I}$ satisfies $r_{ij}(s^{\mathcal{I}_i}) = s^{\mathcal{I}_j}$ if only if it satisfies $r_{ji}(s^{\mathcal{I}_j}) = s^{\mathcal{I}_i}$, for any symbol s such that $P_i \xrightarrow{s} P_j$ (Definition 18 and Proposition 19 in [15]) it must follow that a P-DL ontology can be reduced to an equivalent imports-free ontology. Then, a shared symbol s of P_i and P_j must have the same interpretation from the point of view of both P_i and P_j, i.e., $s^{\mathcal{I}_i} = s^{\mathcal{I}_j}$. However, according to our definition of model (Definition 1), it is *not* the case that a P-DL model $\mathcal{I}$ satisfies $r_{ij}(s^{\mathcal{I}_i}) = s^{\mathcal{I}_j}$ if only if it satisfies $r_{ji}(s^{\mathcal{I}_j}) = s^{\mathcal{I}_i}$, for any symbol s such that $P_i \xrightarrow{s} P_j$. As noted by Bao et al. [2, 3]:

- P-DL semantics does not require the existence of both r_{ij} and r_{ji}. Their joint existence is only required when P_i and P_j mutually import one another. Hence, even if $r_{ij}(s^{\mathcal{I}_i}) = s^{\mathcal{I}_j}$, it is possible that the corresponding r_{ji} may not exist in which case $r_{ji}(s^{\mathcal{I}_j})$ is undefined.

[5] We thank Jeff Pan for discussions on this issue.

- Domain relations are *not necessarily* total functions. Hence, it need not be the case that *every* individual of $\Delta^{\mathcal{I}_i}$ is mapped (by the one-to-one domain relation r_{ij}) to an individual of $\Delta^{\mathcal{I}_j}$.
- Satisfiability and consistency have only contextualized meaning in P-DL. If P_j is not in P_i^*, then models of P_i^* need not be models of P_j^*. This is made clear in Definition 2 where satisfiability and consistency are always considered from the point of view of a witness package.

In the following subsection, we will present an additional example (Example 8) that illustrates the directionality of importing in P-DL.

13.5.4 P-DL Consistency and TBox Consistency

In Section 13.3 we have shown how to reduce a $\mathcal{SHOIQP}$ P-DL ontology to a corresponding DL ($\mathcal{SHOIQ}$) ontology. We have further shown (Theorem 1) that determining the consistency of a $\mathcal{SHOIQP}$ ontology from the point of view of a package P_w can be reduced to the satisfiability of a $\mathcal{SHOIQ}$ concept with respect to a $\mathcal{SHOIQ}$ ontology obtained by integrating the packages imported by P_w. However, it is important to note that this reduction of $\mathcal{SHOIQP}$ is *different* from a reduction based on S-compatibility as defined in [15].

Definition 3 (Expansion). [15] *Let A-interpretation denote an interpretation over a signature A. An S-interpretation $\mathcal{J} = (\Delta^{\mathcal{J}}, \cdot^{\mathcal{J}})$ is an expansion of an S'-interpretation $\mathcal{J}' = (\Delta^{\mathcal{J}'}, \cdot^{\mathcal{J}'})$ if*

(1) $S' \subseteq S$,
(2) $\Delta^{\mathcal{J}'} \subseteq \Delta^{\mathcal{J}}$, *and*
(3) $s^{\mathcal{J}} = s^{\mathcal{J}'}$, *for every* $s \in S'$.

Definition 4 (S-compatibility). [15] *Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be TBoxes expressed in a description logic $\mathcal{L}$, and let S be the shared part of their signatures. We say that $\mathcal{T}_1$ and $\mathcal{T}_2$ are S-compatible if there exists an S-interpretation $\mathcal{J}$, that can be expanded to a model $\mathcal{J}_1$ of $\mathcal{T}_1$ and to a model $\mathcal{J}_2$ of $\mathcal{T}_2$.*

As the following example illustrates, a P-DL ontology is not always reducible to the imports-free ontology that is obtained by simply taking the union of the modules (packages).

Example 8. *Let $\mathcal{T}_1 = \{D \sqcup \neg D \sqsubseteq C\}$, $\mathcal{T}_2 = \{C \sqsubseteq \bot\}$. The shared signature $S = \{C\}$ and $\mathcal{T}_1$ and $\mathcal{T}_2$ are not S-compatible. However, suppose we have a* P-DL *ontology such that $\mathcal{T}_1 \xrightarrow{C} \mathcal{T}_2$ and negation in $\mathcal{T}_1$ becomes contextualized negation $\neg_1$. Then we have a model:*

$$\Delta_1 = C^{\mathcal{I}_1} = D^{\mathcal{I}_1} = \{x\}$$
$$\Delta_2 = \{y\}, C^{\mathcal{I}_2} = \emptyset$$
$$r_{12} = r_{21} = \emptyset$$

On the other hand, all models of a P-DL *ontology where* $\mathcal{T}_2 \xrightarrow{C} \mathcal{T}_1$ *have empty* Δ_1*. Thus, the whole ontology is consistent as witnessed by* $\mathcal{T}_2$ *but inconsistent as witnessed by* $\mathcal{T}_1$*. This example demonstrates that* P-DL *importing is directional.*

The next example shows that, in the presence of nominals, the P-DL consistency problem is not reducible to the consistency of an imports-free ontology obtained by simply combining the P-DL modules.

Example 9 (Use of Nominals). *Consider the following TBoxes:*

$$\mathcal{T}_1 = \{\top \sqsubseteq i \sqcup j, \quad i \sqcap j \sqsubseteq \bot\}$$
$$\mathcal{T}_2 = \{\top \sqsubseteq i\},$$

with the shared signature $S = \{i\}$*, where* i, j *are nominals.* $\mathcal{T}_1$ *and* $\mathcal{T}_2$ *are* S*-compatible but* $\mathcal{T}_1 \cup \mathcal{T}_2$ *is not consistent. Suppose we have a* P-DL *ontology with* $\mathcal{T}_1 \xrightarrow{i} \mathcal{T}_2$*. Since "*$\top$*" only has contextualized meaning in* P-DL*, these TBoxes in fact should be represented as*

$$\mathcal{T}_1 = \{\top_1 \sqsubseteq i \sqcup j, \quad i \sqcap j \sqsubseteq \bot\}$$
$$\mathcal{T}_2 = \{\top_2 \sqsubseteq i\}$$

Now, there exists a model for this P-DL *ontology:*

$$\Delta_1 = \{x, y\}, i^{\mathcal{I}_1} = \{x\}, j^{\mathcal{I}_1} = \{y\}$$
$$\Delta_2 = \{x'\}, i^{\mathcal{I}_2} = \{x'\}$$
$$r_{12} = \{(x, x')\}$$

In general, the reduction from P-DL modules to imports-free TBoxes with shared signatures based on S-compatibility, as suggested by [15], does not preserve the semantics of P-DL. Thus, there is a fundamental difference between the two settings: P-DL has no universal top concept and, as a result, P-DL axioms have only localized effect. In the case of imports-free TBoxes, in the absence of contextualized semantics, it is not possible to ensure that the effects of axioms are localized. Consequently, it is not possible to reduce reasoning with a P-DL ontology with modules $\{\mathcal{T}_i\}$ to standard DL reasoning over the union of all ontology modules $\mathcal{T} = \mathcal{T}_1 \cup ... \cup \mathcal{T}_n$.

In contrast, in the previous section we have shown that such a reduction from reasoning in P-DL from the point of view of a witness package to reasoning with a suitably constructed DL (as shown in Section 13.3) is possible. Nevertheless, relying on such a reduction is not attractive in practice, because it requires the integration of the ontology modules, which may be prohibitively expensive. More importantly, in many scenarios encountered in practice, e.g., in peer-to-peer applications, centralized reasoning with an integrated ontology is simply infeasible. Hence, work in progress is aimed at developing federated reasoners for P-DL that do not require the integration of different ontology modules (see, e.g., [4]).

13.6 Summary

In this chapter, we have introduced a modular ontology language, package-based description logic $\mathcal{SHOIQP}$, that allows reuse of knowledge from multiple ontologies. A $\mathcal{SHOIQP}$ ontology consists of multiple ontology modules each of which can be viewed as a $\mathcal{SHOIQ}$ ontology. Concept, role and nominal names can be shared by "importing" relations among modules.

The proposed language supports contextualized interpretation, i.e., interpretation from the point of view of a specific package. We have established a minimal set of constraints on domain relations, i.e., the relations between individuals in different local domains, that allow the preservation of the satisfiability of concept expressions, the monotonicity of inference, and the transitive reuse of knowledge.

Ongoing work is aimed at developing a distributed reasoning algorithm for $\mathcal{SHOIQP}$ by extending the results of [4] and [20], as well as an OWL extension capturing the syntax of $\mathcal{SHOIQP}$. We are also exploring several variants of P-DL, based on a more in-depth analysis of the properties of the domain relations and the preservation of satisfiability of concept subsumptions across modules.

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