DS 310 Machine Learning Vasant Honavar Fall 2018

Problemset 1: Math for Machine Learning

Available: August 20, 2018 Due: August 27, 2018

- 1. (20 pts.) Let $\mathbf{X} = [2, -2]$ be a vector.
 - (a) Plot the vector in a 2-dimensional Euclidean space.
 - (b) Compute the Euclidian norm of the vector **X**.
 - (c) Normalize the vector **X** to obtain a vector of unit length.
 - (d) Compute the vector that is normal to **X**.
 - (e) Compute the transpose of the vector **X**.
- 2. (20 pts.) Joe takes a lab test and the result comes back positive. The test returns a correct positive result in only 98% of the cases in which the disease is actually present, and a correct negative result in only 97% of the cases in which the disease is not present. Furthermore, .008 of the entire population have this cancer. Does Joe have cancer? Justify your answer.
- 3. (20 pts.) Consider the function $f(x, y) = x^2 + xy + y^2$.
 - (a) Compute the first order partial derivatives ∂fx and ∂fy
 - (b) Compute the Hessian (matrix of second order partial derivatives) of f.
 - (c) Is f(x, y) a convex function of x and y? Justify your answer.
 - (d) Compute the values of x and y at which f(x, y) is minimized.
- 4. (20 pts.) Suppose a coin is tossed. The observed outcomes are H,T,H,H,H,T,H,H,T,H,H,T,H,H,T (where H denotes a head and T denotes a tail).
 - (a) What is the probability of heads? What is the probability of tails?
 - (b) What assumptions did you make in arriving at your answer?
- 5. (20 pts.) Suppose we have a coin that can remember the outcome of its previous toss. Suppose the first toss has equal probability of being a head or a tail. On each subsequent toss, if the current toss was a head, the next toss will be a tail with probability 1/4 and head with probability 3/4. If the current toss was a tail, the next toss will be a head with probability 1/2 and tail with probability 1/2. What is the probability of observing the sequence H,H,T,H?