





# Algorithmic Abstractions of Rational Decision-Making







#### Decision-Theoretic Agent



- Different states have different utility to the agent
- Utility function of a simple decision theoretic agent maps each state onto a real number (utility)
- Actions are chosen based on the expected utility of the resulting state
- More general setting involves making complex decisions involving sequences of actions







#### Making Simple Decisions - Example

#### Decision time for Agent Joe Six Pack









### Combining Beliefs and Desires

- Rational Behavior
  - Based on beliefs about the world, in particular, consequences of one's actions in a given state
  - Must cope with uncertainty
    - There is no way to know for sure the outcome of an action because of partial ignorance or inherently stochastic effects of actions
  - Must provide a means of comparing alternatives using a common currency
    - Partying on a Thursday night versus getting an A







- Bob is contemplating whether he should take insurance on a shipment from Amsterdam to St. Petersburg.
- If the ship does not encounter a storm, the shipment arrives on time in St. Petersburg, and Bob will earn 10,000 rubles
- If the ship encounters a storm, the shipment will be delayed and Bob will earn only 8000 rubles.
- The Amsterdam underwriters want Bob to pay 1000 rubles for a full coverage insurance policy
- Should Bob buy the policy?







- Potential loss to Bob in the event of delayed arrival of shipment = 2000 rubles
- Cost of insurance policy = 1000 rubles
- Should Bob buy the policy?







- Depends
- On what?
  - Probability of storm-related delay
  - If the storm is highly unlikely (say probability of storm ≈ 0.2) at that time of the year, perhaps not
  - If the storm is likely (say probability of storm  $\approx$  0.8) then perhaps
  - Can we translate this intuition into a precise prescription for decision making under uncertainty?







- Suppose Bob believes that the probability of a storm related delay  $\approx 0.2$
- Bob's expected earnings in the absence of insurance
  - = (0.2)(8000) + 0.8 (10,000)
  - = 1600 + 8000
  - = 9600 rubles
- Bob's expected earnings if he purchases insurance
  - = 10,000 1000
  - = 9000 rubles
- Bob is perhaps better off without insurance than with it.







- Suppose Bob believes that the probability of a storm related delay  $\approx 0.8$
- Bob's expected earnings in the absence of insurance
  - = (0.8)(8000) + 0.2(10,000)
  - = 6400 + 2000
  - = 8400 rubles
- Bob's expected earnings if he purchases insurance
  - = 10,000 1000
  - = 9000 rubles
- Bob is better off with insurance than without it.







- St. Petersburg Paradox
- From Nicolas Bernoulli's letter
- Consider the following game
  - Peter flips a fair coin repeatedly until a head shows up and will give Paul:
    - \$2 if the first *head* shows up on the 1<sup>st</sup> flip
    - \$2<sup>2</sup> if the first *head* shows up on the 2<sup>nd</sup> flip
    - ....
    - \$2<sup>k</sup> if the first head shows up on the k<sup>th</sup> flip
  - How much should Paul pay Peter to play this game?







### St. Petersburg Paradox (cont)

$$E(payoff) = \sum_{k=1}^{k=\infty} 2^k \left(\frac{1}{2}\right)^k = 1 + 1 + \dots = \infty$$

- The expected payoff is infinite
- Does this mean it is rational for Paul to pay Peter any finite amount (say \$1 million) to play this game?







### St Petersburg Paradox

- How much should Paul be willing to pay Peter for a chance to play the game?
- Expected payoff = infinity
- But.. There is a risk of loss
  - Suppose Paul pays \$1,000,000 to pay the game
  - Suppose the first head shows up on the 2<sup>nd</sup> toss
  - Paul will receive \$4 and lose \$999996
- Is the gamble worth the risk?
- Depends..







### St Petersburg paradox

- Is Paul's gamble worth the risk of losing almost \$1,000,000?
- Depends
- On what?
  - How much money Paul has to start with
  - The risk might be unacceptable if Paul's entire life savings is \$1,000,000
  - The risk might be perfectly reasonable if Paul has billions in the bank
  - If Paul is poor, he may be justified in paying no more than two dollars, the minimum possible pay-off of the game







### Making simple decisions

- Can we turn the previous intuition into a recipe for decision making under uncertainty?
- Von Neumann Morgenstern solution
  - Maximum Expected Utility (MEU) principle
    - Choose actions that maximize expected utility of outcome
  - As evident from the St. Petersburg paradox, for most people, the *utility* of money is not a linear function of the amount of money







### Making simple decisions

- Notation
  - *U*(*S*) : utility of state *S*
  - S : snapshot of the world
  - A : action of the agent
  - $Result_i(A)$  : *i*th outcome of (state resulting from doing) A
  - E : available evidence
  - Do(A) : executing A in current state S







### Making simple decisions

- Utility function
  - assigns a single number to each outcome
  - models the desirability of the state to an agent
  - combined with probability of each outcome resulting from an action yields expected utility for action leading to each outcome









### Making Decisions

• Expected Utility

$$EU(A | E) = \sum_{i} P(Result_{i}(A) | E, Do(A))U(Result_{i}(A))$$

- Maximum Expected Utility(MEU)
  - Choose an action which maximizes agent's expected utility
- Computing  $P(Result_i(A) | E, Do(A))$  requires a probabilistic model of the world (Bayes Network)
- Computing the utility of a state U(Result<sub>i</sub>(A)) may require search because it can be hard to tell how good a state is until we know where it would lead us







### Decision Theoretic Agent

function DT-AGENT(percept) returns an action
static: belief\_state, probabilistic beliefs about the current state of the world
action, the agent's action

update belief\_state based on action and percept calculate outcome probabilities for actions, given action descriptions and current belief\_state select action with highest expected utility given probabilities of outcomes and utility information return action







#### Lotteries

Lotteries are used to model decision making scenarios

Lotteries have a finite set of possible mutually exclusive outcomes and probabilities associated with each outcome

Simple Lotteries : L = [p, A; 1-p, B] (two outcomes)

$$L = [p_1, A_1; p_2, A_2; \dots p_n, A_n] \text{ (n outcomes)}$$
$$L = [1, A] = A \text{ (1 outcome)}$$

Compound lotteries: outcomes are themselves lotteries  $L_1 = [p, A; 1-p, L_2]$  $L_2 = [p_1, A; p_2, B; p_3, C]$ 







#### Preferences

- $A \succ B$  : A is preferred to B
- $A \sim B$ : indifference between A & B

 $A \succeq B$ : A is preferred to B or there is indifference

between A and B

In general, outcomes such as A, B can be simple outcomes or outcomes of lotteries L = [p, A; 1-p, B]







### **Constraints on Rational Preferences I**

Rational preferences must satisfy "reasonable" constraints on rational behavior

• Orderability  $(A \succ B) \oplus (B \succ A) \oplus (A \thicksim B)$ 

 $\oplus$  denotes exclusive OR

• Transitivity  $(A \succ B) \land (B \succ C) \Longrightarrow (A \succ C)$ 

• Continuity  $A \succ B \succ C \Longrightarrow \exists p \ [p,A; \ 1-p,C] \sim B$ 







### **Constraints on Rational Preferences II**

• Substitutability

$$A \sim B \Longrightarrow [p, A; 1-p, C] \sim [p, B; 1-p, C]$$

Monotonicity

$$A \succ B \Longrightarrow (p > q \Leftrightarrow [p, A; 1 - p, B] \succ [q, A; 1 - q, B]$$
$$A \sim B \Longrightarrow (p = q \Leftrightarrow [p, A; 1 - p, B] \sim [q, A; 1 - q, B]$$

Decomposability

 $[p,A;\!1-p,\![q,B;\!1-q,C]] \sim [p,A;(1-p)q,B;(1-p)(1-q),C]$ 







# **Utility Function**

- Utility Function models an agent's preferences
- Utility principle

 $U(A) > U(B) \Leftrightarrow A \succ B$ 

$$U(A) = U(B) \Leftrightarrow A \sim B$$

• Expected Utility of a lottery

$$U([p_1, x_1; ...; p_n, x_n]) = \sum_i p_i U(x_i)$$

• Utility principle applies to lotteries

$$[p_1, x_1; \ldots; p_n, x_n] \succ [q_1, x_1; \ldots; q_n, x_n] \Leftrightarrow \sum_i p_i U(x_i) > \sum_i q_i U(x_i)$$







#### Existence of an utility function I

- Consider a simple lottery  $l = [p_1, x_1; p_2, x_2; ..., p_r, x_r]$ where  $x_1 \succ x_2 \succ \cdots x_{r-1} \succ x_r$
- The continuity axiom guarantees the existence of  $u_i$ that ensures indifference between each prize  $x_i$ and a reference lottery  $[u_i, x_1; (1-u_i), x_r]$ Hence, we have

$$l = [p_1, x_1; p_2, x_2; ... p_r, x_r]$$
  
~  $[p_1, [u_1, x_1; (1 - u_1), x_r]; p_2, [u_2, x_1; (1 - u_2), x_r]... p_r, [u_r, x_1; (1 - u_r), x_r]]$ 







Existence of an utility function II

Note that each reference lottery  $[u_i, x_1; (1-u_i), x_r]$ 

is equivalent to a simple lottery:

$$[u_i, x_1; 0, x_2; 0, x_3; ..0, x_{r-1}; (1-u_i), x_r]$$

Using the decomposability axiom, we have:

$$l = [p_1, x_1; p_2, x_2; ... p_r, x_r]$$
  

$$\sim [p_1, [u_1, x_1; (1-u_1), x_r]; p_2, [u_2, x_1; (1-u_2), x_r] ... p_r, [u_r, x_1; (1-u_r), x_r]]$$
  

$$\sim [(p_1u_1 + p_2u_2 + ... + p_ru_r), x_1; (p_1(1-u_1) + p_2(1-u_2) + ... + p_r(1-u_r)), x_r]$$
  

$$\sim [\left(\sum_{i=1}^r p_i u_i\right), x_1; \left(\sum_{i=1}^r p_i(1-u_i)\right), x_r]$$







#### Existence of an utility function III

#### We showed

$$l = [p_1, x_1; p_2, x_2; \dots p_r, x_r] \sim \left[ \left( \sum_{i=1}^r p_i u_i \right), x_1; \left( \sum_{i=1}^r p_i (1 - u_i) \right), x_r \right]$$

$$l' = [q_1, x_1; q_2, x_2; \dots q_r, x_r] \sim \left[ \left( \sum_{i=1}^r q_i u_i \right), x_1; \left( \sum_{i=1}^r q_i (1 - u_i) \right), x_r \right]$$







#### Existence of an utility function IV

Orderability and monotonicity axioms imply

$$\begin{split} l \succ l' \Leftrightarrow \left[ \left( \sum_{i=1}^{r} p_{i} u_{i} \right), x_{1}; \left( \sum_{i=1}^{r} p_{i} (1-u_{i}) \right), x_{r} \right] \succ \left[ \left( \sum_{i=1}^{r} q_{i} u_{i} \right), x_{1}; \left( \sum_{i=1}^{r} q_{i} (1-u_{i}) \right), x_{r} \right] \\ \Leftrightarrow \sum_{i=1}^{r} p_{i} u_{i} \geq \sum_{i=1}^{r} q_{i} u_{i} \\ l \sim l' \Leftrightarrow \left[ \left( \sum_{i=1}^{r} p_{i} u_{i} \right), x_{1}; \left( \sum_{i=1}^{r} p_{i} (1-u_{i}) \right), x_{r} \right] \sim \left[ \left( \sum_{i=1}^{r} q_{i} u_{i} \right), x_{1}; \left( \sum_{i=1}^{r} q_{i} (1-u_{i}) \right), x_{r} \right] \\ \Leftrightarrow \sum_{i=1}^{r} p_{i} u_{i} = \sum_{i=1}^{r} q_{i} u_{i} \end{split}$$

Setting  $u_i = U(x_i)$ , we have established that the existence of a utility function follows from the constraints on rational preferences







### **Eliciting Utility Function**

Need to map states or outcomes to real numbers

- Compare A to standard lottery
  - utility of the best possible prize with probability p
    - *u* =1 :
  - utility of the worst possible catastrophe with prob. 1-p

L

• *u* =0

• Adjust p until  $A \sim L_p$  U(A) = p









### Some remarks on utility functions

- Constraints on rational preferences do not guarantee a unique utility function
  - positive linear transformation will not change preferences

$$U'(x) = k_1 U(x) + k_2$$
 where  $k_1 > 0$ 

• Utility of money is typically not a linear function of money









U

0

#### **Utility Functions**

- Given a lottery **L**
- risk-averse  $U(S_L) < U(S_{EMV(L)})$
- risk-seeking

 $U(S_L) > U(S_{EMV(L)}) \setminus$ 



\$





# Multi-attribute Utility function

- Multi-Attribute Utility Theory (MAUT)
  - Outcomes are characterized by 2 or more attributes.
  - E.g., choice of an automobile might need to take into account
    - Price of the automobile
    - Fuel economy
    - Safety
    - •
- Approach
  - Identify regularities in the preferences to simplify decision-making







### **Multi-attribute Utility Function**

- Notation
  - Attributes  $X_1, X_2, X_3, \ldots$
  - Attribute value vector  $X = < x_1, x_2, \dots >$
  - Utility Function

 $U(x_1,\ldots,x_n)$ 







### Multi-attribute Utility Theory

- Dominance
  - Certain (strict dominance, Fig.1)
    - eg) airport site  $S_1$  costs less, leads to less noise, safer than  $S_2$ 
      - : strict dominance of  $S_1$  over  $S_2$
  - Uncertain (Fig. 2)





**Computational Foundations of Informatics** 





### Multi-attribute Utility Theory

Stochastic dominance more common in real-world settings

- $S_1$ : avg \$3.7billion, standard deviation : \$0.4billion
- $S_2$ : avg \$4.0billion, standard deviation : \$0.35billion
  - S<sub>1</sub> stochastically dominates S<sub>2</sub>







#### Multi-attribute Utility Theory

Stochastic dominance









# Multi-attribute Utility Function

Consider first preferences in the absence of uncertainty

- preferences between concrete outcome values.
- Preference structure
  - $X_1 \& X_2$  *preferentially independent* of  $X_3$  if preference between  $< x_1, x_2, x_3 > \& < x'_1, x'_2, x_3 >$

does not depend on the particular choice of  $\chi_2$ 

Example: Airport site location: <Noise,Cost,Safety>

- Suppose Noise and Cost are preferentially independent of Safety:
- If <20,000 suffer, \$4.6billion, 0.06deaths/mpm is preferred over</li>
   <70,000 suffer, \$4.2billion, 0.06deaths/mpm> then
   <20,000 suffer, \$4.6billion, 0.08deaths/mpm> is preferred over
   <70,000 suffer, \$4.2billion, 0.08deaths/mpm>






#### Multi-attribute Utility Function

- Preferences without Uncertainty
- Mutual preferential independence (MPI)
  - Each pair of attributes is preferentially independent of the rest
  - eg) Airport site : <Noise, Cost, Safety>
    - Noise & Cost P.I Safety
    - Noise & Safety P.I Cost
    - Cost & Safety P.I Noise
      - : <Noise,Cost,Safety> exhibits MPI

m

Agent's preference behavior

$$\max\left[V(S) = \sum_{I} (V_{i}X_{i}(S))\right]$$







Multi-attribute Utility Functions

- Preferences in the presence of uncertainty
  - Preferences between Lotteries
  - Utility Independence (UI)
    - X is utility-independent of Y iff preferences over lotteries' attribute set X do not depend on particular values of a set of attribute Y.
  - Mutual Utility Independence (MUI)
    - Each subset of attributes is UI of the remaining attributes.
  - Mutual Utility Independence (MUI) implies a multiplicative utility function
  - Example (3 MUI attributes)

 $U = k_1U_1 + k_2U_2 + k_3U_3 + k_1k_2U_1U_2 + k_2k_3U_2U_3 + k_3k_1U_3U_1 + k_1k_2k_3U_1U_2U_3$ 







#### **Decision Networks**

- Simple formalism for expressing and solving decision problems
- Bayesian networks + decision & utility nodes
- Nodes
  - Chance nodes
  - Decision nodes
  - Utility nodes







#### A Simple Decision Network









#### A Simplified representation









#### **Evaluating Decision Networks**

Function DN-eval (percept) returns action

static *D*, a decision network

set evidence variables for the current state for each possible value of decision node set decision node to that value calculate Posterior Prob. For parent nodes of the utility node calculate resulting utility for the action select the *action* with the highest utility

return action







#### Value of Information

- Information reduces uncertainty
- Information improves quality of decisions
- How to assess the value of information?
- Example: Buying rights for a diamond mine
  - Three blocks A, B and C, exactly one has diamonds, worth \$K
  - Prior probability that any one block has diamonds= 1/3
  - Current price of each block is K/3
  - Consultant offers results of a survey that definitively indicates whether or not block A contains diamonds
  - How much should you pay for the results of the survey?







#### What would you do if you had the results of the survey?

- With probability 1/3, the survey will indicate diamonds in block A
  - Buy block A
  - Profit = K (K/3) = (2K/3) \$
- With probability 2/3, the survey will indicate no diamonds in block A
  - Buy a block other than A
  - Probability of finding diamonds in one of the remaining blocks
    (B,C) = <sup>1</sup>/<sub>2</sub>
  - Expected profit = (*K*/2) (*K*/3) = (*K*/6) \$







What would you do if you had the results of the survey?

Expected profit given the survey information

$$= \left(\frac{1}{3}\right)\left(\frac{2K}{3}\right) + \left(\frac{2}{3}\right)\left(\frac{K}{6}\right) = \left(\frac{K}{3}\right)\$$$

Expected profit in the absence of the survey information

$$= \left(\frac{1}{3}\right) K - \left(\frac{K}{3}\right) = 0$$

 $\therefore$  The survey results for block A are worth at most  $\left(\frac{K}{3}\right)$ \$







#### Value of Information

- Expected value of Information
  - = Expected value of best action given information
    - Expected value of best action without information

• Survey results "diamonds in A" or 'no diamonds in A"

$$= \frac{1}{3} \times \frac{k}{3} + \frac{2}{3} \times \left(\frac{1}{2} \times \frac{k}{3} + \frac{1}{2} \times \frac{k}{3}\right) - 0 = \frac{k}{3}$$







#### General recipe for assessing the value of information

- Current evidence E, Current best action  $\boldsymbol{\alpha}$
- Possible action outcomes Result<sub>i</sub>(A)=S<sub>i</sub>
- Potential new evidence *E<sub>i</sub>*
- Expected utility EU
- Value of perfect information

$$EU(\alpha \mid E) = \max_{A} \sum_{i} U(S_{i}) P(S_{i} \mid E, Do(A))$$
$$EU(\alpha_{E_{j}} \mid E, E_{j}) = \max_{A} \sum_{i} U(S_{i}) P(S_{i} \mid E, Do(A), E_{j})$$
$$VPI_{E}(E_{j}) = \left(\sum_{k} P(E_{j} = e_{jk} \mid E) EU(\alpha_{e_{jk}} \mid E, E_{j} = e_{jk})\right) - EU(\alpha \mid E)$$







#### Properties of Value of Perfect Information

- Nonnegative  $\forall j, E \quad VPI_E(E_j) \ge 0$
- Nonadditive

$$VPI_{E}(E_{j}, E_{k}) \neq VPI_{E}(E_{j}) + VPI_{E_{j}}(E_{k})$$

• Order-Independent

 $VPI_{E}(E_{j}, E_{k}) = VPI_{E}(E_{j}) + VPI_{E, E_{j}}(E_{k}) = VPI_{E}(E_{k}) + VPI_{E, E_{k}}(E_{j})$ 

 Imperfect information about a variable X can be modeled by perfect information about a variable Y that is probabilistically related to variable X







#### Information Gathering Agent

function INFORMATION-GATHERING-AGENT(percept) returns an action static: D, a decision network

integrate percept into D  $j \leftarrow$  the value that maximizes  $VPI(E_j) - Cost(E_j)$ if  $VPI(E_j) > Cost(E_j)$ then return REQUEST( $E_j$ ) else return the best action from D







#### Making Simple Decisions: Summary

- Utility theory offers a prescriptive framework for rational decision-making under uncertainty
- Bounded rationality:
  - In the real world, decisions often have to be made with imperfect information, and under tight time and resource constraints
  - Resource bounded rationality often relies on simple heuristics that make us smart







#### Representing & Reasoning with Qualitative Preferences







## Outline

- I. Qualitative Preference Languages
  - Representation : Syntax of languages CP-nets, TCP-nets, CI-nets, CP-Theories
- II. Qualitative Preference Languages
  - Ceteris Paribus semantics: the induced preference graph (IPG)
  - Reasoning: Consistency, Dominance, Ordering, Equivalence & Subsumption
  - Complexity of Reasoning
- III. Practical aspects: Preference Reasoning via Model Checking
  - From ceteris paribus semantics (IPG) to Kripke structures
  - Specifying and verifying properties in temporal logic
  - Translating Reasoning Tasks into Temporal Logic Properties







## **Decision Theory**

- What is a *decision*?
  - Choosing from a set of *alternatives A*
- How are alternatives described?
  - What influences choice of an agent?
    - preferences, uncertainty, risk
  - Can decisions be automated?
  - What happens if there are multiple agents?
    - conflicting preferences and choices

"I prefer walking over driving to work"

There is a 50% chance of snow. Walking may not be good after all.







## **Qualitative Preferences**









#### Representation: Alternatives are Multiattributed



- Preference variables or attributes used to describe the domain
- Alternatives are assignments to preference variables
  - $\alpha$  = (instructor = Gopal, area = AI, credits = 3)
- $\alpha > \beta$  denotes that  $\alpha$  is preferred to  $\beta$







#### Qualitative Preference Languages

#### Qualitative preferences

- Unconditional Preferences
  - TUP-nets [Santhanam et al., 2010]
- Conditional Preferences
  - CP-nets [Boutilier et al. 1997,2002]
  - Models dependencies
- Relative Importance
  - TCP-nets [Brafman et al. 2006]
  - CI-nets [Bouveret et al. 2009]

$\succ_{area} SE$	
	Γ.

SE ∶Tom ≻ <sub>instructor</sub> Gopal	
AI : Gopal $\succ_{instructor}$ Tom	

			1
Instructor	$\triangleright$	Credits	ļ









*E=Functional:* Unavailable > Official fix







- Conditional Preference Table (CPT) annotates nodes
- CPT can be partially specified
- Comparative preferences over:
  - Pairs of values of an attribute
  - Pairs of attributes (importance)















- Conditional Importance Networks (Cl-nets) [Bouveret 2009] Cl-nets (fair division of goods among agents)
- Preference variables represent items to be included in a deal
- Preference variables are Binary (presence/absence of an item)
- Intra-variable Preference is monotonic (0 > 1 or 1 > 0)
  - Subsets preferred to supersets (or vice versa) by default
- CI-net Statements are of the form  $S^+$ ,  $S^- : S_1 > S_2$ 
  - Represents preference on the presence of one set of items over another set under certain conditions
  - If all propositions in S<sup>+</sup> are true and all propositions in S<sup>-</sup> are false, then the set of propositions S<sub>1</sub> is preferred to S<sub>2</sub>







#### Conditional Importance Networks (CI-nets) [Bouveret 2009]

**Cl-nets** (fair division of goods among agents)

- a = Name
- b = Address
- c = Bank Routing Number
- d = Bank Account Number

P1. C } D P2. а P3. {}, {d} : {a, b}  $\succ$  {c}

#### If I have to ...

disclose my address without having to disclose my name,

then I would prefer ...

giving my bank routing number

over ...

my bank account number







#### Other Preference Languages

- Preference languages in Databases [Chomicki 2004]
- Preferences over Sets [Brafman et al. 2006]
- Preferences among sets (incremental improvement)<sup>[Brewka et al. 2010]</sup>
- Tradeoff-enhanced Unconditional Preferences (TUP-nets) [Santhanam et al. 2010]
- Cardinality-constrained Cl-nets (C<sup>3</sup>I-nets) [Santhanam et al. 2013]







#### Relative Expressivity of Preference Languages

Preferences over Multi-domain Variables

Preferences over (Sets of) Binary Variables











# Preference Reasoning Exact Reasoning about Qualitative Preferences Not covered :

- Uncertainty + Preferences
  - Cornelio et al. Updates and Uncertainty in CP-Nets 2013
  - Bigot et al. *Probabilistic CP-nets 2013*
- Applications
  - Rossi et al. Preference Aggregation: Social Choice 2012
  - Chomicki et al. Skyline queries in Databases 2011
  - Trabelsi et al. Preference Induction Recommender systems 2013
- Other Reasoning Approaches
  - Minyi et al. *Heuristic approach to dominance testing in CP-nets* 2011
  - Wilson Upper Approximation for Conditional Preferences 2006







#### Other Preference Languages

- Preference languages in Databases [Chomicki 2004]
- Preferences over Sets [Brafman et al. 2006]
- Preferences among sets (incremental improvement)<sup>[Brewka et al. 2010]</sup>
- Tradeoff-enhanced Unconditional Preferences (TUP-nets) [Santhanam et al. 2010]
- Cardinality-constrained CI-nets (C<sup>3</sup>I-nets) [Santhanam et al. 2013]
- We limit our discussion to CP-nets, TCP-nets and CI-nets
- Approach extensible to all other preference languages with Ceteris paribus semantics







#### Key concepts

- Induced Preference Graph (IPG)
- Semantics in terms of flips in the IPG
- Reasoning Tasks
  - Dominance over Alternatives
  - Equivalence & Subsumption of Preferences
  - Ordering of Alternatives
- Complexity of Reasoning







## Induced Preference Graph (IPG) [Boutilier et al. 2001]

- *Induced preference graph*  $\delta(P) = G(V, E)$  of preference spec P:
  - Nodes V : set of alternatives



- δ(N) is acyclic (dominance is a strict partial order)
- $\alpha > \beta$  iff there is a *path* in  $\delta(N)$  from  $\beta$  to  $\alpha$  (serves as the *proof*)

Santhanam et al. AAAI 2010







#### Preference Semantics in terms of IPG

- $(\beta, \alpha) \in E$  iff there is a *flip* from  $\alpha$  to  $\beta$  *"induced by some preference" in P*
- Types of flips
  - Ceteris Paribus flip flip a variable, "all other variables equal"
  - Specialized flips
    - Relative Importance flip
    - Set based Importance flip
    - Cardinality based Importance flip
- Languages differ in the semantics depending on the specific types of flips they allow

... Next:

examples







Flips for a CP-net [Boutilier et al. 2001]

•  $(\beta, \alpha) \in E$  iff there is a statement in CP-net such that  $x_1 >_1 x'_1$  ( $x_1$  is preferred to  $x'_1$ ) and ...

- V-flip : all other variables being equal,  $\alpha(X_1)=x_1$  and  $\beta(X_1)=x'_1$ 



<u>Single</u> variable flip – change value of 1 variable at a time







#### Flips for TCP-nets & CP-theories [Brafman et al., Wilson 2004]

- (α, β) ∈ E iff there is a statement in TCP-net such that x<sub>1</sub> ><sub>1</sub> x'<sub>1</sub> (x<sub>1</sub> is preferred to x'<sub>1</sub>) and ...
  - V-flip : all other variables being equal,  $\alpha(X_1)=x_1$  and  $\beta(X_1)=x'_1$
  - *I-flip* : all variables *except* <u>those</u> *less important than*  $X_1$  *being equal*,  $\alpha(X_1)=x_1$  and  $\beta(X_1)=x'_1$









## Flips for a Cl-net [Bouveret 2009]

- CI-nets express *preferences over subsets* of binary variables X.
  - Truth values of X<sub>i</sub> tells its presence/absence in a set
  - Nodes in IPG correspond to subsets of X
  - Supersets are always preferred to Strict Subsets (convention)
  - $S^+$ ,  $S^- : S_1 > S_2$  interpreted as ...

If all propositions in S<sup>+</sup> are true and all propositions in S<sup>-</sup> are false, then the set of propositions  $S_1$  is preferred to  $S_2$ 

- For  $\alpha$ ,  $\beta \subseteq X$ ,  $(\alpha, \beta) \in E$  ( $\beta$  preferred to  $\alpha$ ) iff
  - *M-flip* : all other variables being equal,  $\alpha \subset \beta$
  - **CI-flip** : there is a CI-net statement s.t.  $S^+$ ,  $S^- : S_1 > S_2$  and  $\alpha$ ,  $\beta$  satisfy  $S^+$ ,  $S^-$  and  $\alpha$  satisfies  $S^+$  and  $\beta$  satisfies  $S^-$ .







# Flips for a Cl-net [Bouveret 2009]

- For  $\alpha$ ,  $\beta \subseteq X$ ,  $(\alpha, \beta) \in E$  ( $\beta$  preferred to  $\alpha$ ) iff
  - *M-flip* : all other variables being equal,  $\alpha \subset \beta$
  - **CI-flip** : there is a CI-net statement  $S^+$ ,  $S^- : S_1 > S_2$  s.t.  $\alpha$ ,  $\beta$  satisfy  $S^+$ ,  $S^-$  and  $\alpha$  satisfies  $S^+$  and  $\beta$  satisfies  $S^-$
- Example:
  - a = Name
  - b = Address
  - c = Bank Routing Number
  - d = Bank Account Number

P1.  $\{d\}, \{\} : \{b\} \succ \{c\}$ P2.  $\{b\}, \{a\} : \{c\} \succ \{d\}$ P3. {}, {d} : {a, b} > {c}



#### Oster et al. FACS 2012




- Flips for a C<sup>3</sup>I-net [Santhanam et al. 2013]
- C<sup>3</sup>I-nets express *preference over subsets* similar to CI-net
  - Truth values of X<sub>i</sub> tells its presence/absence in a set
  - Nodes in IPG correspond to subsets of X
  - Sets with *higher cardinality* are preferred (conventional)
  - $S^+$ ,  $S^- : S_1 > S_2$  interpreted as ...

If all propositions in S<sup>+</sup> are true and all propositions in S<sup>-</sup> are false, then the set of propositions  $S_1$  is preferred to  $S_2$ 

• For  $\alpha$ ,  $\beta \subseteq X$ ,  $(\alpha, \beta) \in E$  ( $\beta$  preferred to  $\alpha$ ) iff

- *M-flip* : all other variables being equal,  $|\alpha| < |\beta|$ 

- CI-flip : there is a CI-net statement s.t. S<sup>+</sup>, S<sup>-</sup> : S<sub>1</sub> > S<sub>2</sub> and α, β satisfy S<sup>+</sup>, S<sup>-</sup> and α satisfies S<sup>+</sup> and β satisfies S<sup>-</sup>.
- Extra cardinality constraint to enable dominance







## Flips for a C<sup>3</sup>I-net [Santhanam et al. 2013]

- For  $\alpha$ ,  $\beta \subseteq X$ ,  $(\alpha, \beta) \in E$  ( $\beta$  preferred to  $\alpha$ ) iff
  - *M*-flip :  $\alpha \subset \beta$  (all other variables being equal)
  - *CI-flip* : there is a CI-net statement S<sup>+</sup>, S<sup>-</sup> : S<sub>1</sub> > S<sub>2</sub> s.t.  $\alpha$ ,  $\beta$  satisfy S<sup>+</sup>, S<sup>-</sup> and  $\alpha$  satisfies S<sup>+</sup> and  $\beta$  satisfies S<sup>-</sup>.







## **Reasoning Tasks**

The semantics of any ceteris paribus language can be represented in terms of properties of IPG

- Now we turn to the Reasoning Tasks:
  - Dominance & Consistency
  - Equivalence & Subsumption
  - Ordering
- Reasoning tasks reduce to verifying properties of IPG







- **Dominance** relation: Reasoning Tasks  $\alpha > \beta$  iff there exists a sequence of flips from  $\beta$  to  $\alpha$
- Property to verify: *Existence of path in IPG* from  $\beta$  to  $\alpha$ Consistency:
- A set of preferences is *consistent* if > is a strict partial order
- Property to verify: *IPG is acyclic*
- semantics

Equivalence (& Subsumption):

- A set P<sub>1</sub> of preferences is *equivalent* to another set P<sub>2</sub> if they induce the same dominance relation
- Property to verify: IPGs are reachability equivalent







## **Reasoning Tasks**

Reasoning Task	Computation Strategy: Property of IPG to check	Remarks
Dominance: $\alpha > \beta$	Is $\beta$ reachable from $\alpha$ ?	
Consistency of a set of preferences (P)	Is the IPG of P acyclic?	Satisfiability of the dominance relation; strict partial order
Equivalence of two sets of preferences P <sub>1</sub> and P <sub>2</sub>	Are the IPGs of P <sub>1</sub> and P <sub>2</sub> reachability-equivalent?	
Subsumption of one set of preference ( $P_1$ ) by another ( $P_2$ )	If $\beta$ reachable from $\alpha$ in the IPG of P <sub>1</sub> , does the same hold in the IPG of P <sub>2</sub> ?	
Ordering of alternatives	Iterative verification of the IPG for the non-existence of the non-dominated alternatives	Iterative modification of the IPG to obtain next set of non-dominated alternatives







**PSPACE-complete** 

## Complexity of Dominance [Goldsmith et al. 2008] Cast as a *search* for a flipping sequence, or *a path in IPG*



- α = (A = 1, B = 0, C = 0)
- β = (A = 0, B = 1, C = 1)
- $\alpha > \beta Why$ ?







# Complexity of Reasoning Tasks

Reasoning Task	Complexity	Source
Dominance: $\alpha > \beta$	PSPACE-complete	Goldsmith et al. 2008
Consistency of a set of preferences (P)	PSPACE-complete	Goldsmith et al. 2008
Equivalence of two sets of preferences $P_1$ and $P_2$	PSPACE-complete	Santhanam et al. 2013
Subsumption of one set of preference $(P_1)$ by another $(P_2)$	PSPACE-complete	Santhanam et al. 2013
Ordering of alternatives	NP-hard	Brafman et al. 2011







## **Practical Aspects**

Part III – Outline

- Two Sound and Complete Reasoning Approaches:
  - Logic Programming
    - Answer Set Programming [Brewka et al.]
    - Constraint Programming [Brafman et al. & Rossi et al. ]
  - Model Checking based
    - Preference reasoning can be reduced to verifying properties of the IPG [Santhanam et al. 2010]
    - Translate IPG into a Kripke Structure Model
    - Translate reasoning tasks into temporal logic properties over model
- Approximation & Heuristics
  - Wilson [Wilson 2006, 2011]







# Preference Reasoning via Model Checking

- The *first practical solution to preference reasoning* in moderate sized CP-nets, TCP-nets, CI-nets, etc.
  - Casts dominance testing as reachability in an induced graph
  - Employs direct, succinct encoding of preferences using Kripke structures
  - Uses Temporal logic (CTL, LTL) for querying Kripke structures
  - Uses direct translation from reasoning tasks to CTL/LTL
    - Dominance Testing
    - Consistency checking (loop checking using LTL)
    - Equivalence and Subsumption Testing
    - Ordering (next-preferred) alternatives

Santhanam et al. (AAAI 2010, KR 2010, ADT 2013); Oster et al. (ASE 2011, FACS 2012)







 $\varphi$ 

# Model Checking <sup>[Clark et al. 1986]</sup>

- Model Checking: Given a desired property, (typically expressed as a temporal logic formula), and a (Kripke) structure M with initial state s, decide if M, s ⊨
- Active area of research in formal methods, AI (SAT solvers)
- Broad range of applications: hardware and software verification, security..
- Temporal logic languages : CTL, LTL, μ-calculus, etc.
- Many model checkers available : SMV, NuSMV, Spin, etc.

Advantages of Model Checking:

- 1. Formal Guarantees
- 2. Justification of Results







# Preference Reasoning via Model Checking

Preference reasoning can be reduced to verifying properties of the Induced Preference Graph [Santhanam et al. 2010]

#### Overview of Approach

- 1. Translate IPG into a Kripke Structure Model
- 2. Translate reasoning tasks into verification of temporal logic properties on the model







### **Overview: Preference Reasoning via Model Checking**







# Kripke Structure [Kripke, 1963]

A Kripke structure is a 4-tuple  $K=(S, S_0, T, L)$  over variables V, where

- S represents the set of reachable states of the sy
- *S*<sub>0</sub> is a set of initial states
- *T* represents the set of state transitions

Used to specify labeled transition systems describing states of the world w.r.t. flow of time

• L is labeling (interpretation) function maps each node to a set of atomic propositions AP that hold in the corresponding state

Computational tree temporal logic (CTL) is an extension of propositional logic

 Includes temporal connectives that allow specification of properties that hold over states and paths in K

Example

• *EF* true in state *s* of *K* if holds in some state in some path beginning at *s* 







# **Encoding Preference Semantics**

Let P = {p<sub>i</sub>} be a set of ceteris paribus preference statements on a set of preference variables X = { $x_1, x_2, ...$ }

Reasoning Strategy:

- Construct a Kripke model  $K_P = (S, S_0, T, L)$  using variables Z
  - $Z = \{z_i \mid x_i \in X\}$ , with each variable  $z_i$  having same domain  $D_i$  as  $x_i$
  - K<sub>P</sub> must mimic the IPG in some sense
- The State-Space of K<sub>P</sub>
  - S =  $\prod_{i \in D_i}$ : states correspond to set of all alternatives
  - T : transitions correspond to allowed changes in valuations according to flip-semantics of the language
  - L : labeling (interpretation) function maps each node to a set of atomic propositions AP that hold in the corresponding state
  - $-S_0$ : Initial states assigned according to the reasoning task at hand







#### From Syntax to Semantics Encode K<sub>P</sub> such that paths in IPG are enabled transitions, and no

additional transitions are enabled

- Let p be a conditional preference statement in P
- p induces a flip between two nodes in the IPG iff
  - 1. "Condition" part in the preference statement is satisfied by both nodes
  - 2. "Preference" part (less & more preferred valuations) is satisfied by both
  - 3. "Ceteris Paribus" part that ensures apart from (1 & 2) that all variables other than those specified to change as per (2) are equal in both nodes
- Create transitions in K<sub>P</sub> with guard conditions
  - "Condition" part of statement is translated to the guard condition
  - "Preference" part of statement is translated to assignments of variables in the target state
  - How to ensure ceteris paribus condition?







# From Syntax to Semantics

Encode  $K_P$  such that paths in IPG are enabled transitions, and no additional transitions are enabled

- Let p be a conditional preference statement in P
- p induces a flip between two nodes in the IPG iff
  - 1. "Condition" part in the preference statement is satisfied by both nodes
  - 2. "Preference" part (less & more preferred valuations) in satisfied by both
  - 3. "Ceteris Paribus" part that ensures apart from (1 & 2) that all variables other than those specified to change as per (2) are equal in both nodes
- Create transitions in K<sub>P</sub> with guard conditions
  - "Condition" part of statement is translated to the guard condition
  - "Preference" part of statement is translated to assignments of variables in the target state

How to encode ceteris paribus condition in the guards?







# From Syntax to Semantics

<u>Recall</u>: In temporal logics, destination states represent "future" state of the world

- Equality of source and destination states forbidden as part of the guard condition specification!
- Workaround: Use auxiliary variables h<sub>i</sub> to label edges

 $h_{i} = \begin{cases} 0 \Rightarrow \text{value of } z_{i} \text{ must not change in a} \\ \text{transition in the Kripke structure } K(P) \\ 1 \Rightarrow \text{otherwise} \end{cases}$ (1)

• Auxiliary edge labels don't contribute to the state space







# From Syntax to Semantics

Guard condition specification

- <u>Recall:</u> p induces a flip between two nodes in the IPG iff
  - 1. "Condition" part in the preference statement is satisfied by both nodes
  - 2. "Preference" part (less & more preferred valuations) in satisfied by both
  - 3. "Ceteris Paribus" part that ensures apart from (1 & 2) that all variables other than those specified to change as per (2) are equal in both nodes
- For each statement p of the form  $\varrho: x_i = v_i \succ_{x_i} x_i = v'_i$ where  $\varrho$  is the "condition" part, guard condition is  $\mathcal{G}(p) = Allow(p) \land Restrict(p) \text{ s.t. condition}$  preference





**Computational Foundations of Informatics** 

ollege of Information





# **Encoding CP-net semantics**









llege of Information



ollege of Information





### Encoding Reasoning Tasks as Temporal Logic Properties

Next :

Specifying and Verifying Properties in Temporal Logic

Translating Reasoning Tasks into Temporal Logic Properties







# Encoding Reasoning Tasks as Temporal Logic Computational tree temporal logic (CTL) <sup>[Clark et al. 1986]</sup> is an extension

- of propositional logic
- Includes temporal connectives that allow specification of properties that hold over states and paths in a Kripke structure
- CTL Syntax & Semantics

**EX**  $\psi$  if there exists a path  $s = s_1 \rightarrow s_2 \dots$  such that  $s_2$  satisfies  $\psi$ 

AX  $\psi$  if for all paths such that  $s = s_1 \rightarrow s_2 \dots, s_2$  satisfies  $\psi$ 

 $\mathbb{E} [\psi_1 \mathbb{U} \ \psi_2]$  if there exists a path  $s = s_1 \rightarrow s_2 \dots$  such that  $\exists i \geq 1 : s_i$  satisfies  $\psi_2$ , and  $\forall j < i : s_j$  satisfies  $\psi_1$ 

- Translating Reasoning Tasks into Temporal Logic Properties
  - Dominance Testing
  - Consistency
  - Equivalence & Subsumption Testing
  - Ordering alternatives

NuSMV <sup>[Cimatti et al. 2001]</sup>: Our choice of model checker





# Dominance Testing (via NuSMV)

Given outcomes  $\alpha$  and  $\beta$ , how to check if  $\alpha > \beta$ ?

- Let  $\phi_{\alpha}$  be a formula that holds in the state corresponding to  $\alpha$
- Let  $\phi_\beta$  be a formula that holds in the state corresponding to  $\beta$

By construction,  $\alpha > \beta$  wrt iff in the Kripke Structure  $K_N$ :

#### a state in which $\phi_{\beta}$ holds is reachable from a state in which $\phi_{\alpha}$ holds

- $\alpha \succ \beta$  iff the model checker NuSMV can verify  $\varphi_{\alpha} \rightarrow EF \varphi_{\beta}$  (SAT)
- When queried with  $\neg($   $\varphi_{\alpha} \rightarrow EF\varphi_{\beta}$ ), if indeed  $\alpha > \beta$ , then model checker produces a proof of  $\alpha > \beta$  (flipping sequence)
- Experiments show feasibility of method for 100 var. in seconds















#### **Obtaining a Proof of Dominance** $(a = 1 \land b = 0 \land c = 0)$ $\Rightarrow EF(a = 0 \land b = 1 \land c = 1)$

• 011 is preferred to 100 *Improving* flipping sequence:  $100 \rightarrow 101 \rightarrow 001 \rightarrow 000 \rightarrow 011$ 











## Non-dominance

• 011 is not preferred to 000









# Equivalence and Subsumpt. Combined Induced Preference Graph







te from which verification is done

ollege of Information tiences And Technology

$$\varphi : \mathbf{AX} \left( g_1 \Rightarrow \mathbf{EX} \ \mathbf{E} \left[ g_2 \ \mathbf{U} \ (\psi \land g_2) \right] \right) \qquad \neg \varphi : \mathbf{EX} \left( g_1 \land \mathbf{AX} \ \neg \mathbf{E} \left[ g_2 \ \mathbf{U} \ (\psi \land g_2) \right] \right)$$
$$\operatorname{True} \Leftrightarrow \mathsf{P}_1 \sqsubseteq \mathsf{P}_2 \qquad \qquad \mathsf{False} \Leftrightarrow \mathsf{P}_2 \nvDash \mathsf{P}_1$$
$$\operatorname{Santhanam et al. ADT 2013} \qquad \mathsf{Model Checker returns} \qquad \overline{ab} \ ; a\overline{b} \operatorname{roof}$$







# Ordering : Finding the Next-preferred Alternative

- Which alternatives are most-preferred (non-dominated)?
- Can we enumerate all alternatives in order?



We verify a sequence of reachability properties encoded in CTL

Acyclic Case: Oster et al. FACS 2012







# Applications

- Sustainable Design of Civil Infrastructure (e.g., Buildings, Pavements)
- Engineering Design (Aerospace, Mechanical)
- Strategic & mission critical decision making (Public policy, Defense, Security)
- Site Selection for Nuclear Waste and setting up new nuclear plants
- Software Engineering
  - Semantic Search
  - Code Search, Search based SE
  - Program Synthesis, Optimization
  - Test prioritization
  - Requirements Engineering
- Databases Skyline queries
- Stable Marriage problems
- Al Planning, configuration
- Recommender Systems





Center for Big Data Analytics and Discovery Informatics



Sustainable Design







Center for Big Data Analytics and Discovery Informatics





Table 1: Available Building Components in the Repository







Goal Oriented Requirements Engineering

Oster et al. ASE 2011






**Center for Big Data Analytics and Discovery Informatics Artificial Intelligence Research Laboratory** 



#### Goal oriented Requirements **Engineering** – CI-nets





AND

Provide

Quote

Books Ordered

ΔND

Supplier

Provides

Price

Contact

Supplier

Customer

Requests

Quote





### Applications - Minimizing Credential Disclosure

Oster et al. FACS 2012

- User needs renter's insurance for new apartment
  - Which service to choose to get a quote?
  - Privacy issue disclosure of sensitive credentials
- All services do the same tasks (from user's perspective) info:

	#	Name	Required Sensitive Information
	1	QuickQuote	Address, Bank Account #
	2	InsureBest	Name, Address, Bank Routing #
	3	EZCoverage	Name, Address
r	4	BankMatch	Bank Routing #

User's Prefer

P1. If bank account number is disclosed, then I would rather give my address than bank routing number to the server

P2. If I have to disclose my address but not my name, then I would prefer to give my bank routing number rather than my bank account number

P3. If I don't need to disclose my bank account number, I will give my name and address instead of my bank routing number.







## Applications - Minimizing Credential Disclosure

Oster et al. FACS 2012

- Finding a sequence of next-preferred
  - Subantimal coquance of proferred cats of cradentials
- a = Name
- b = Address
- c = Bank Routing Number
- d = Bank Account Number

P1.  $\{d\}, \{\}$  :  $\{b\} \succ \{c\}$ P2.  $\{b\}, \{a\}$  :  $\{c\} \succ \{d\}$ P3.  $\{\}, \{d\}$  :  $\{a, b\} \succ \{c\}$ 









#### **CRISNER** Preference Reasoning Tool

- CRISNER freely available at
  - http://www.ece.iastate.edu/~gsanthan/crisner.html
- Currently supports representing and reasoning with
  - CI-nets
  - CP-nets
- Reasoning tasks supported
  - Dominance Testing
  - Consistency
  - Next-preferred (for acyclic CP/CI-nets)
  - Support for Equivalence & Subsumption testing coming







#### **CRISNER** Architecture

- Architecture decouples preference reasoning from choice of
  - Model checker
  - Translation of preference
  - Preference languages
  - Modular design enables extension to other ceteris paribus languages, reasoning tasks and encodings
- Tool Dependencies
  - Model Checker NuSMV or Cadence SMV
  - Java Runtime Environment





Center for Big Data Analytics and Discovery Informatics Artificial Intelligence Research Laboratory



### CRISNER Architecture

Preference Reasoning tasks

(dominance/consistency/ordering/equivalence)







Center for Big Data Analytics and Discovery Informatics Artificial Intelligence Research Laboratory



### **CRISNER** Architecture

Tool Dependencies

- Model Checker NuSMV or Cadence SMV
- Java Runtime Environment

Input/Output

- Preference specifications encoded in XML
  - Translated to SMV (Kripke model encoding)
- Parsers to translate output of model checker
- Iterative process to compute alternatives in order





Ι.



### Summary

- Qualitative Preference Languages
  - Representation : Syntax of languages CP-nets, TCP-nets, CI-nets, CP-Theories
- II. Qualitative Preference Languages
  - Ceteris Paribus semantics: the induced preference graph (IPG)
  - Reasoning: Consistency, Dominance, Ordering, Equivalence & Subsumption
  - Complexity of Reasoning
- III. Practical aspects: Preference Reasoning via Model Checking
  - From ceteris paribus semantics (IPG) to Kripke structures
  - Specifying and verifying properties in temporal logic
  - Reasoning tasks reduce to verification of temporal properties







# Summary

- **IV.** Applications
  - Engineering: Civil, Software (SBSE, RE, Services), Aerospace, Manufacturing
  - Security: Credential disclosure, Cyber-security
  - Algorithms: Search, Stable Marriage, Allocation, Planning, Recommender systems
  - Environmental applications: Risk Assessment, Policy decisions, Environmental impact, Computational Sustainability

#### V. CRISNER

- A general, practically useful Preference Reasoner for ceteris paribus languages
- Architecture
- Use of CRISNER in Security, Software Engineering

