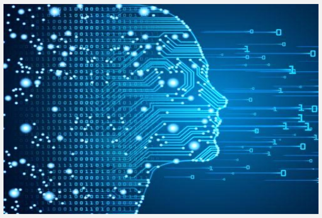

Principles of Causal Inference

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Plan for the rest of the semester

- Selected Advanced topics
  - Advanced methods for learning causal models
  - Causal models of relational data, temporal data
  - Causal effects generalizability (e.g., transportability)
- Selected Applications
  - Healthcare
  - Algorithmic fairness
  - Explainable ML
- Student project presentations

Causal transportability[1]

- Suppose we have run a study in Chicago and learned a causal relationship, say between poverty and obesity
- Suppose we want to see if the relationship is true in some form in Los Angeles
  - Los Angeles is different from Chicago in some respects, e.g., demographics
- We now have tools to answer if the causal relationship which we learned from a study in Chicago can be tweaked in some way so that it applies to Los Angeles

[1]Bareinboim and Pearl, 2012; Lee and Honavar, 2013a; 2013b, Bareinboim, Lee, Honavar, and Pearl, 2013, Bareinboim and Pearl, 2016; Lee et al., 2019.

Transportability of Causal Effects Across Populations

Source Population Π

Target Population Π^*

$P(y, t, x \cdots)$

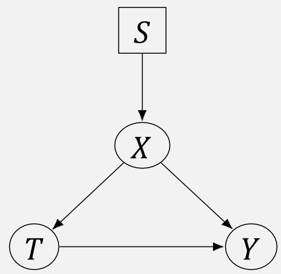
$P^*(y, t, x \cdots)$

Given $P(y \mid do(t), x)$ $\quad P(y \mid do(t), x) \stackrel{?}{=} P^*(y \mid do(t), x)$

Selection Diagrams

Selection diagrams

- Allow for different causal mechanisms across the source and target distributions (Π and Π^*)

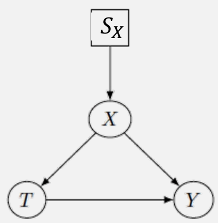


$$P^*(y \mid do(t), x) \triangleq P(y \mid do(t), x, s^*)$$

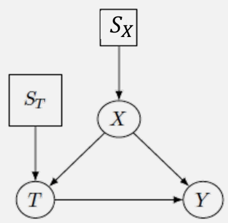
Selection Diagrams

Selection diagrams

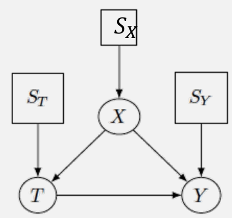
- Allow for different causal mechanisms across the source and target distributions (Π and Π^*)



Π_1^*

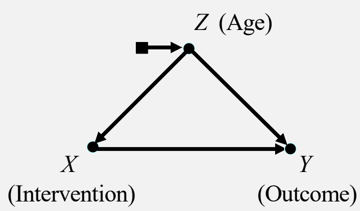


Π_2^*



Π_3^*

Causal transportability



- Source Π and target Π^* differ with respect to the distribution of Z
- For example, Π includes all ages, Π^* includes only young
- Indicated by the "selection" arrow into Z

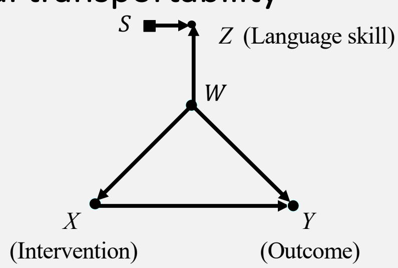
Experimental study in LA
Measured: $P(x, y, z), P(y|do(x), z)$

Observational study in NYC
Measured: $P^*(x, y, z)$
$P^*(z) \neq P(z)$

Needed:
$Q = P^*(y|do(x)) = \sum_z P(y|do(x), z)P^*(z)$

Transport Formula: $F(P, P_{do}, P^*)$

Causal transportability



- Source Π and target Π^* differ with respect to the distribution of Z
- For example, Π and Π^* differ through “selection” on Z (language skill)

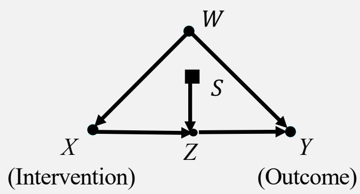
Experimental study in LA
Measured: $P(x, y, w, z), P(y|do(x), w)$
Needed:
$$Q = P^*(y|do(x)) = P(y|do(x))$$

Transport Formula: $F(P, P_{do}, P^*)$

Observational study in NYC
Measured: $P^*(x, y, w, z)$
$$P^*(z) \neq P(z)$$

- Not all differences between source and target matter!

Causal transportability



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Experimental study in LA
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Measured: $P^*(x, y, w, z)$
$P^*(z) \neq P(z)$

Needed:
$Q = P^*(y|do(x)) = P(y|do(x), z)\ P^*(z|x)$

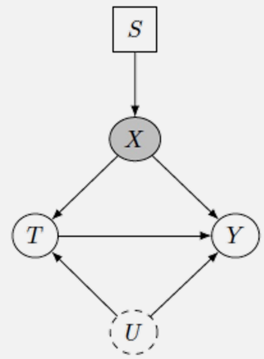
Transport Formula: $F(P, P_{do}, P^*)$

Causal transportability reduced to do-calculus

- Theorem: A causal relation ***R*** is transportable from a source domain Π to a target domain Π^*
- if and only if it is reducible, using the rules of ***do***-calculus, to an expression in which the selection variable(s) ***S*** is(are) separated from ***do***( ).

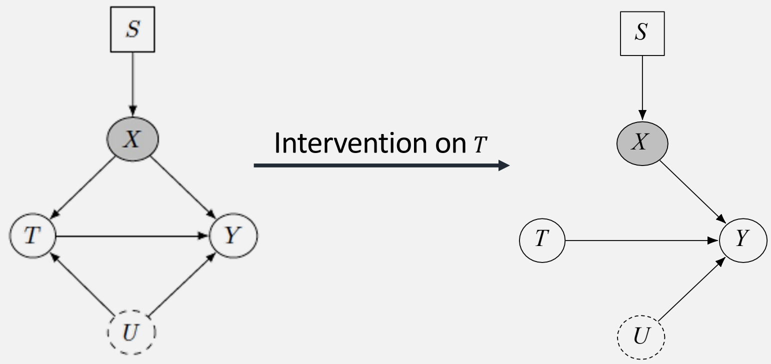
Transportability

$$P(y \mid do(t), x) \stackrel{?}{=} P^*(y \mid do(t), x)$$



Transportability and do-calculus

$$P(y \mid do(t), x) \stackrel{?}{=} P^*(y \mid do(t), x)$$



Proof:

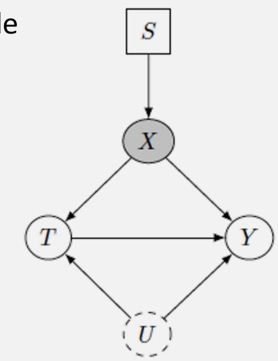
$$P^*(y \mid do(t), x) = P(y \mid do(t), x, s^*)$$
$$= P(y \mid do(t), x)$$

External validity (direct transportability)

- We say that the causal effect $P(y|do(t),x)$ is directly transportable from source domain Π to a target domain Π^* if

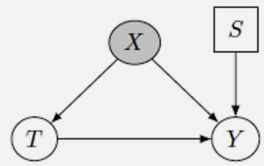
$$P(y \mid do(t), x) = P^*(y \mid do(t), x)$$

- Such a causal effect is said to have external validity (or generalizability)



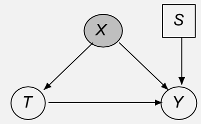
Transportability

$$P(y \mid do(t), x) \stackrel{?}{=} P^*(y \mid do(t), x)$$



$$P(y \mid do(t), x) \neq P^*(y \mid do(t), x)$$

Trivial Transportability



- We clearly don't have direct transportability
  - $P(y \mid do(t), x) \neq P^*(y \mid do(t), x)$
- Suppose we have access to observational data from the target population: $P^*(y, t, x)$
- Then we can identify $P^*(y \mid do(t), x)$ using only target data
  - $P^*(y \mid do(t), x) = P^*(y \mid t, x)$
- If a causal effect is identifiable from observational data in the target domain,
  - We do not need any information from the source domain to estimate it
  - It is trivially transportable from any source domain

S-admissibility and transportability

- A set of variables W is said to be S —admissible if
$$Y \perp\!\!\!\perp_{G_{\overline{T}}} S \mid T, W$$
- If W is S —admissible, then
$$P^*(y \mid do(t)) = \sum_w P(y \mid do(t), w)\, P^*(w)$$

S-admissibility and transportability: Example

- A set of variables W is said to be S —admissible if $Y \perp\!\!\!\perp_{G_{\overline{T}}} S \mid T, W$
- If W is S —admissible, then

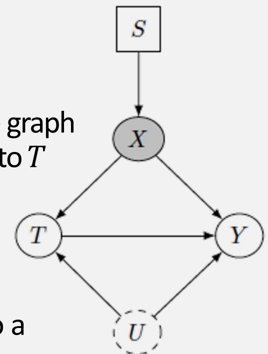
$$P^*(y \mid do(t)) = \sum_w P(y \mid do(t), w)\, P^*(w)$$

S-admissibility and transportability: Example

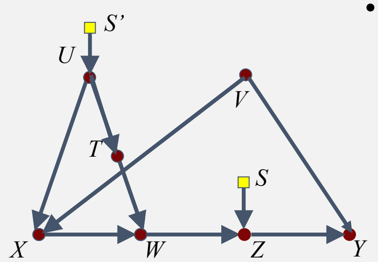
- In the example shown, can you identify a set of variables that is S-admissible?
- Yes, $\{X\}$ is S-admissible
  - Y is d-separated from S given X and T in the graph resulting from removal of incoming edges into T
- Hence,

$$P^*(y|do(t)) = \sum_x P(y|do(t), x)\, P^*(x)$$

- That is, $P^*(y|do(t))$ is transportable from Π to a target domain Π^*



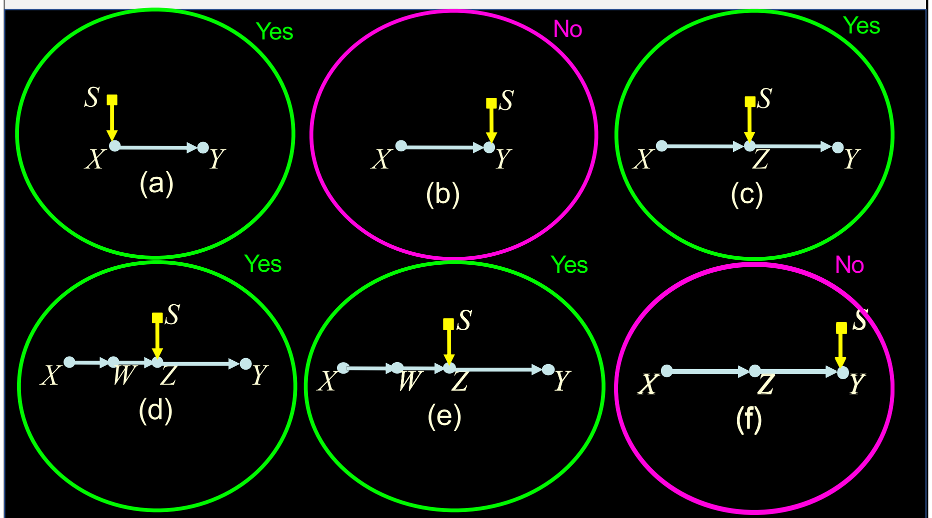
Extercise



- Is $P^*(y|do(x))$ transportable from source to target the setting shown?

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) \sum_w P^*(z \mid w) \sum_t P(w \mid do(w), t) P^*(t)$$

In which of the scenarios the causal effect of X on Y transportable?



Causal transportability – general version

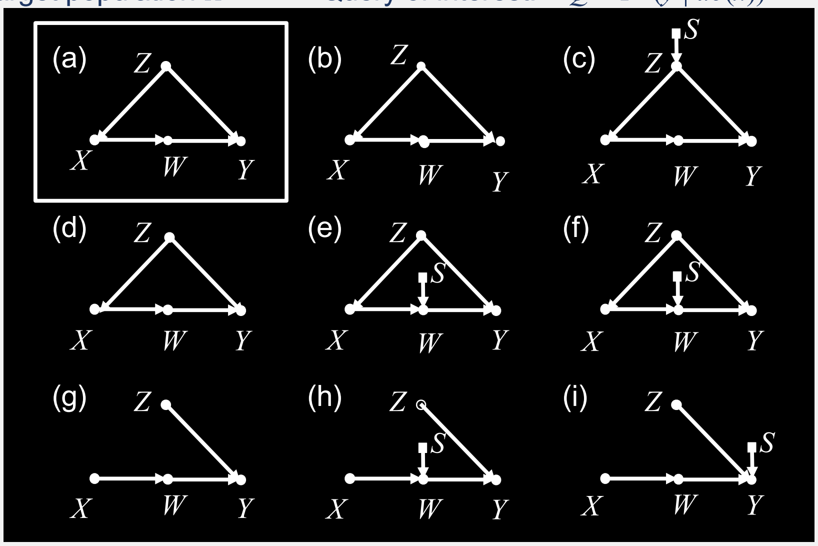
- How to combine results of
  - several experimental and observational studies,
  - each conducted on a different population and under a different set of conditions,
  - to construct a valid estimate a causal effect of interest,
  - in a new (target) population,
  - that may be different from any of the ones studied

Causal transportability

Target population Π^* Query of interest: $Q = P^*(y \mid do(x))$

(a) Arkansas Only survey data	(b) New York Only survey data Resembling target	(c) Los Angeles Only survey data Young fashionistas
(d) Boston Age not recorded Mostly educated scholars	(e) San Francisco Mostly techies	(f) Texas Mostly Hispanics
(g) State College RCT College students	(h) Utah RCT paid volunteers	(i) Wyoming RCT, young athletes

Target population Π^* Query of interest: $Q = P^*(y \mid do(x))$



Summary

- Given the commonalities and differences between one or more source domains and a target domain encoded in selection diagrams, transportability of a causal effect of interest from the source domain(s) to a target domain can be determined using do-calculus
- When an effect is transportable, the transport formula can be derived in time that is polynomial in the size of the formula
- The algorithm is sound and complete
- Corollary do-calculus is complete for causal transportability

Further generalizations

- mz-transportability
  - Identification from proxy experiments
  - Multiple transportability
- Meta analysis

Completeness of do calculus for causal inference

Do calculus is complete for

- ✓Causal transportability
  - Bareinboim & Pearl, 2012
- ✓Causal m-transposability
  - Bareinboim & Pearl, 2013; Lee and Honavar, 2013
- ✓Causal z transposability
  - Bareinboim & Pearl, 2013; Lee & Honavar, 2013
- ✓Causal mz-transportability
  - Bareinboim, Lee, Honavar & Pearl, 2013
- ✓Meta analysis
  - Bareinboim et al., 2016; Lee et al., 2019

Discussion

- Open problems related to transportability and meta analysis