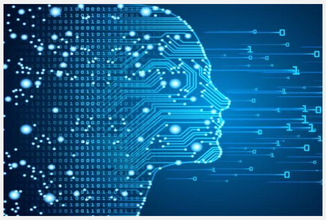

Principles of Causal Inference

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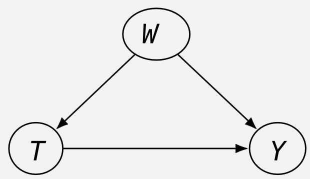
Unobserved Confounding, Bounds, and Sensitivity Analysis

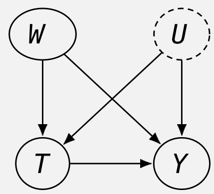
Adjusting for confounders revisited



$$\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}_W\left[\mathbb{E}[Y|T = 1, W] - \mathbb{E}[Y|T = 0, W]\right]$$

Unobserved Confounding

- Suppose U is an unobserved confounder
- We cannot adjust for U



$$\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}_{W,U}\left[\mathbb{E}[Y|T = 1, W, U] - \mathbb{E}[Y|T = 0, W, U]\right]$$

$$\cong \mathbb{E}_{W}\left[\mathbb{E}[Y|T = 1, W] - \mathbb{E}[Y|T = 0, W]\right]?$$

Bounds

- No-Assumptions Bound
- Monotone Treatment Response
- Monotone Treatment Selection
- Optimal Treatment Selection

Sensitivity Analysis

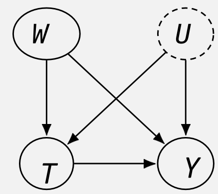
- Linear Single Confounder
- More General Settings

Unobserved Confounding

- The assumption that there is no unobserved confounding is unrealistic in practice
- The credibility of inference decreases with the strength of the assumptions made[1]

[1]Manski, C.F., 2003. *Partial identification of probability distributions* (Vol. 5). New York: Springer.

Unobserved Confounding



- What can we do in the presence of unobserved confounding?
  - Turn a blind eye and get a point estimate of the causal effect of T on Y

$$\mathbb{E}[Y(1) - Y(0)] \cong \mathbb{E}_W [\mathbb{E}[Y|T = 1, W] - \mathbb{E}[Y|T = 0, W]]$$

  - Weaken the assumption that there is no unobserved confounding
    - Bound the causal effect
    - Trivial bound is easy to get
      - Potential outcomes are bounded: $\forall t \; a \leq Y(t) \leq b$
      - $\forall i \; (a - b) \leq Y_i(1) - Y_i(0) \leq (b - a)$
      - $(a - b) \leq \mathbb{E}[Y(1) - Y(0)] \leq (b - a)$
    - Can we do better?

Observational-Counterfactual decomposition

$$\begin{aligned}
\mathbb{E}[Y(1) - Y(0)] &= \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] \\
&= P(T=1)\,\mathbb{E}[Y(1) \mid T=1] + P(T=0)\,\mathbb{E}[Y(1) \mid T=0] \\
&\quad - P(T=1)\,\mathbb{E}[Y(0) \mid T=1] - P(T=0)\,\mathbb{E}[Y(0) \mid T=0] \\
&= P(T=1)\,\mathbb{E}[Y \mid T=1] + P(T=0)\,\mathbb{E}[Y(1) \mid T=0] \\
&\quad - P(T=1)\,\mathbb{E}[Y(0) \mid T=1] - P(T=0)\,\mathbb{E}[Y \mid T=0]
\end{aligned}$$

Observational-Counterfactual decomposition

$$\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$
$$= P(T=1)\,\mathbb{E}[Y(1) \mid T=1] + P(T=0)\,\mathbb{E}[Y(1) \mid T=0]$$
$$- P(T=1)\,\mathbb{E}[Y(0) \mid T=1] - P(T=0)\,\mathbb{E}[Y(0) \mid T=0]$$

Observational $= P(T=1)\,\underline{\mathbb{E}[Y \mid T=1]} + P(T=0)\,\underline{\mathbb{E}[Y(1) \mid T=0]}$

Counterfactual $- P(T=1)\,\underline{\mathbb{E}[Y(0) \mid T=1]} - P(T=0)\,\underline{\mathbb{E}[Y \mid T=0]}$

$$\mathbb{E}[Y(1) - Y(0)] = \pi\,\mathbb{E}[Y \mid T=1] + (1-\pi)\,\mathbb{E}[Y(1) \mid T=0]$$
$$- \pi\,\mathbb{E}[Y(0) \mid T=1] - (1-\pi)\,\mathbb{E}[Y \mid T=0]$$

where $\pi \triangleq P(T=1)$

Observational-Counterfactual decomposition

$$\mathbb{E}[Y(1)-Y(0)] = \pi\,\mathbb{E}[Y \mid T=1] + (1-\pi)\,\mathbb{E}[Y(1) \mid T=0] - \pi\,\mathbb{E}[Y(0) \mid T=1] - (1-\pi)\,\mathbb{E}[Y \mid T=0]$$

where $\pi \triangleq P(T=1)$

Assuming that counterfactual outcomes are bounded

- $\forall t\ a \leq Y(t) \leq b$

We get

- No (causal) assumptions bound on the causal effects

$$\mathbb{E}[Y(1)-Y(0)] \leq \pi\,\mathbb{E}[Y \mid T=1] + (1-\pi)\,b - \pi\,a - (1-\pi)\,\mathbb{E}[Y \mid T=0]$$

$$\mathbb{E}[Y(1)-Y(0)] \geq \pi\,\mathbb{E}[Y \mid T=1] + (1-\pi)\,a - \pi\,b - (1-\pi)\,\mathbb{E}[Y \mid T=0]$$

No assumptions bound compared to the trivial bound

- Trivial bound

$$a - b \leq \mathbb{E}[Y(1) - Y(0)] \leq b - a$$

  - Bounds the effect within an interval of length

$$2(b - a)$$

- No assumptions bound

$$\mathbb{E}[Y(1) - Y(0)] \leq \pi\, \mathbb{E}[Y \mid T = 1] + (1 - \pi)\, b - \pi\, a - (1 - \pi)\, \mathbb{E}[Y \mid T = 0]$$

$$\mathbb{E}[Y(1) - Y(0)] \geq \pi\, \mathbb{E}[Y \mid T = 1] + (1 - \pi)\, a - \pi\, b - (1 - \pi)\, \mathbb{E}[Y \mid T = 0]$$

  - Bounds the effect within an interval of length

$$(1 - \pi)\, b + \pi\, b - \pi\, a - (1 - \pi)\, a$$

Non-negative Monotone Treatment Response

- Suppose the potential outcomes are bounded in $[a, b]$
- Suppose treatment never hurts $\forall i \; Y_i(1) \geq Y_i(0)$
  - Then $\forall i \; \text{ITE}_i \geq 0$
  - That is, ITE lower bound changes from $a - b$ to 0
  - Then it is easy to prove that the ATE lower bound also changes from $a - b$ to 0

Proof (using factual counterfactual decomposition)

$$\begin{aligned}
\mathbb{E}[Y(1) - Y(0)] &= \pi\, \mathbb{E}[Y \mid T=1] + (1-\pi)\, \mathbb{E}[Y(1) \mid T=0] && \text{(Observational-Counterfactual Decomposition)} \\
&\quad - \pi\, \mathbb{E}[Y(0) \mid T=1] - (1-\pi)\, \mathbb{E}[Y \mid T=0] \\
&\geq \pi\, \mathbb{E}[Y \mid T=1] + (1-\pi)\, \mathbb{E}[Y \mid T=0] && \mathbb{E}[Y(1) \mid T=0] \geq \mathbb{E}[Y \mid T=0] \\
&\quad - \pi\, \mathbb{E}[Y \mid T=1] - (1-\pi)\, \mathbb{E}[Y \mid T=0] && -\mathbb{E}[Y(0) \mid T=1] \geq -\mathbb{E}[Y \mid T=1] \\
&= 0
\end{aligned}$$

Example

- Suppose potential outcomes are bounded between 0 and 1
- Suppose $\mathbb{E}[Y|T=1]=0.9$ and $\mathbb{E}[Y|T=0]=0.2$ and $\pi=0.3$
- No assumptions bound

$$-0.17 \leq \mathbb{E}[Y(1)-Y(0)] \leq 0.83$$

- Nonnegative MTR lower bound

$$\mathbb{E}[Y(1)-Y(0)] \geq 0$$

- Combining the two, we have

$$0 \leq \mathbb{E}[Y(1)-Y(0)] \leq 0.83$$

Non-positive Monotone Treatment Response

- Suppose the potential outcomes are bounded in $[a, b]$
- Suppose treatment never helps $\forall i \; Y_i(0) \geq Y_i(1)$
  - Then $\forall i \; \text{ITE}_i \leq 0$
  - That is, ITE upper bound changes from $b - a$ to 0
  - Then it is easy to prove that the ATE upper bound also changes from $b - a$ to 0

Prove that in the case of non-positive treatment response

$$\mathbb{E}[Y(1) - Y(0)] \leq 0$$

Monotone Treatment Selection

- Treatment group's potential outcomes are better than the control group's

$$\mathbb{E}[Y(1) \mid T = 1] \geq \mathbb{E}[Y(1) \mid T = 0]$$
$$\mathbb{E}[Y(0) \mid T = 1] \geq \mathbb{E}[Y(0) \mid T = 0]$$

Under monotone treatment selection, we can prove that the ATE is bounded from the above by the associational difference

$$\mathbb{E}[Y(1) - Y(0)] \leq \mathbb{E}[Y \mid T = 1] - \mathbb{E}[Y \mid T = 0]$$

Monotone Treatment Selection

- Under monotone treatment selection, we can prove that the ATE is bounded from the above by the associational difference

$$\mathbb{E}[Y(1) - Y(0)] \leq \mathbb{E}[Y \mid T = 1] - \mathbb{E}[Y \mid T = 0]$$

Proof: From observational-counterfactual decomposition, we have:

$$\begin{aligned}\mathbb{E}[Y(1) - Y(0)] &= \pi\,\mathbb{E}[Y \mid T = 1] + (1-\pi)\,\mathbb{E}[Y(1) \mid T = 0] \\ &\quad - \pi\,\mathbb{E}[Y(0) \mid T = 1] - (1-\pi)\,\mathbb{E}[Y \mid T = 0] \\ &\leq \pi\,\mathbb{E}[Y \mid T = 1] + (1-\pi)\,\mathbb{E}[Y \mid T = 1] \\ &\quad - \pi\,\mathbb{E}[Y \mid T = 0] - (1-\pi)\,\mathbb{E}[Y \mid T = 0] \\ &= \mathbb{E}[Y \mid T = 1] - \mathbb{E}[Y \mid T = 0]\end{aligned}$$

- Because

$$\mathbb{E}[Y(1) \mid T = 1] \geq \mathbb{E}[Y(1) \mid T = 0]$$
$$\mathbb{E}[Y(0) \mid T = 1] \geq \mathbb{E}[Y(0) \mid T = 0]$$

and

$$\mathbb{E}[Y(1)|T = 1] = \mathbb{E}[Y|T = 1]$$
$$\mathbb{E}[Y(0)|T = 0] = \mathbb{E}[Y|T = 0]$$

Monotone Treatment Selection (MTS), non-negative Monotone Treatment Response (MTR) and No-Assumptions Lower Bound Combined

Potential outcomes bounded between 0 (a) and 1 (b)

$$\pi = 0.3 \qquad \mathbb{E}[Y \mid T = 1] = .9 \qquad \mathbb{E}[Y \mid T = 0] = .2$$

No-assumptions bound: $-0.17 \leq \mathbb{E}[Y(1) - Y(0)] \leq 0.83$

MTS upper bound: $\mathbb{E}[Y(1) - Y(0)] \leq \mathbb{E}[Y \mid T = 1] - \mathbb{E}[Y \mid T = 0]$

Combining MTS upper bound with no-assumptions lower bound:

$$-0.17 \leq \mathbb{E}[Y(1) - Y(0)] \leq 0.7$$

Adding nonnegative MTR assumption and combining MTS upper bound with MTR lower bound ($\mathbb{E}[Y(1) - Y(0)] \geq 0$):

$$0 \leq \mathbb{E}[Y(1) - Y(0)] \leq 0.7$$

Optimal Treatment Selection

- Under optimal treatment selection, individuals get the treatment that is best for them

$$T_i = 1 \implies Y_i(1) \geq Y_i(0)$$
$$T_i = 0 \implies Y_i(0) > Y_i(1)$$

- Under optimal treatment selection, we can prove that

$$\mathbb{E}[Y(1) - Y(0)] < \pi\, \mathbb{E}[Y \mid T = 1] - \pi\, a$$
$$\mathbb{E}[Y(1) - Y(0)] \geq (1 - \pi)\, a - (1 - \pi)\, \mathbb{E}[Y \mid T = 0]$$

Optimal Treatment Selection

- Under optimal treatment selection $\mathbb{E}[Y(1) \mid T=0] \leq \mathbb{E}[Y(0) \mid T=0] = \mathbb{E}[Y \mid T=0]$
  $\mathbb{E}[Y(0) \mid T=1] \leq \mathbb{E}[Y(1) \mid T=1] = \mathbb{E}[Y \mid T=1]$
- From the no assumptions bound, we have

$$\mathbb{E}[Y(1)-Y(0)] = \pi\,\mathbb{E}[Y \mid T=1] + (1-\pi)\,\mathbb{E}[Y(1) \mid T=0] - \pi\,\mathbb{E}[Y(0) \mid T=1] - (1-\pi)\,\mathbb{E}[Y \mid T=0]$$

Upper bound

$$\begin{aligned} &\leq \pi\,\mathbb{E}[Y \mid T=1] + (1-\pi)\,\mathbb{E}[Y \mid T=0] \\ &\quad - \pi\,a - (1-\pi)\,\mathbb{E}[Y \mid T=0] \\ &= \pi\,\mathbb{E}[Y \mid T=1] - \pi\,a \end{aligned}$$

Lower bound

$$\begin{aligned} &\geq \pi\,\mathbb{E}[Y \mid T=1] + (1-\pi)\,a \\ &\quad - \pi\,\mathbb{E}[Y \mid T=1] - (1-\pi)\,\mathbb{E}[Y \mid T=0] \\ &= (1-\pi)\,a - (1-\pi)\,\mathbb{E}[Y \mid T=0] \end{aligned}$$

Optimal Treatment Selection Bound

$$\mathbb{E}[Y(1) - Y(0)] < \pi\,\mathbb{E}[Y \mid T = 1] - \pi\,a$$
$$\mathbb{E}[Y(1) - Y(0)] \geq (1-\pi)\,a - (1-\pi)\,\mathbb{E}[Y \mid T = 0]$$
$$\text{Interval Length} = \pi\,\mathbb{E}[Y \mid T = 1] + (1-\pi)\,\mathbb{E}[Y \mid T = 0] - a$$

Example

Potential outcomes bounded between 0 (a) and 1 (b)

$$\pi = 0.3 \qquad \mathbb{E}[Y \mid T = 1] = .9 \qquad \mathbb{E}[Y \mid T = 0] = .2$$

No-assumptions bound: $-0.17 \leq \mathbb{E}[Y(1) - Y(0)] \leq 0.83$

OTS Bound 1: $-0.14 \leq \mathbb{E}[Y(1) - Y(0)] \leq 0.27$

Interval Length $= 0.41$

OTS Bound on the Sign of the Effect

- OTS implies $T_i = 1 \implies Y_i(1) \geq Y_i(0)\,, \quad T_i = 0 \implies Y_i(0) > Y_i(1)$
- Hence, we have

$$\begin{aligned}\mathbb{E}[Y(1) \mid T = 0] &= \mathbb{E}[Y(1) \mid Y(0) > Y(1)] \\ &\leq \mathbb{E}[Y(1) \mid Y(0) \leq Y(1)] \\ &= \mathbb{E}[Y(1) \mid T = 1] \\ &= \mathbb{E}[Y \mid T = 1]\end{aligned}$$

- Observational-counterfactual decomposition

$$\begin{aligned}\mathbb{E}[Y(1) - Y(0)] &= \pi\, \mathbb{E}[Y \mid T = 1] + (1-\pi)\, \mathbb{E}[Y(1) \mid T = 0] \\ &\quad - \pi\, \mathbb{E}[Y(0) \mid T = 1] - (1-\pi)\, \mathbb{E}[Y \mid T = 0] \\ &\leq \pi\, \mathbb{E}[Y \mid T = 1] + (1-\pi)\, \mathbb{E}[Y \mid T = 1] \\ &\quad - \pi\, a - (1-\pi)\, \mathbb{E}[Y \mid T = 0] \\ &= \mathbb{E}[Y \mid T = 1] - \pi\, a - (1-\pi)\, \mathbb{E}[Y \mid T = 0]\end{aligned}$$

OTS Bound on the Sign of the Effect

- OTS implies $T_i = 1 \implies Y_i(1) \geq Y_i(0)\,, \quad T_i = 0 \implies Y_i(0) > Y_i(1)$
- Under OTS, we proved an upper bound

$$\mathbb{E}[Y(1) - Y(0)] \leq \mathbb{E}[Y \mid T = 1] - \pi\, a - (1 - \pi)\, \mathbb{E}[Y \mid T = 0]$$

- Analogously, we can prove a lower bound

$$\mathbb{E}[Y(1) - Y(0)] \geq \pi\, \mathbb{E}[Y \mid T = 1] + (1 - \pi)\, a - \mathbb{E}[Y \mid T = 0]$$

Example

$$\pi = 0.3 \qquad \mathbb{E}[Y \mid T = 1] = .9 \qquad \mathbb{E}[Y \mid T = 0] = .2$$

Under OTS the above bound dictates

$$0.07 \leq \mathbb{E}[Y(1) - Y(0)] \leq 0.76$$

Thus establishing the sign of the effect (positive in this case)

Combining OTS bounds

OTS Bound 1: $-0.14 \leq \mathbb{E}[Y(1) - Y(0)] \leq 0.27$

Interval Length $= 0.41$

OTS Bound 2: $0.07 \leq \mathbb{E}[Y(1) - Y(0)] \leq 0.76$

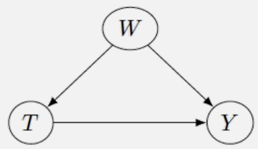
Interval Length $= 0.69$

OTS Upper Bound 1 and OTS Lower Bound 2:

$$0.07 \leq \mathbb{E}[Y(1) - Y(0)] \leq 0.27$$ Interval

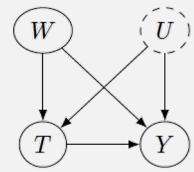
Sensitivity Analysis

- The previous bounds made no assumptions regarding confounders



$$\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}_W \left[\mathbb{E}[Y \mid T = 1, W] - \mathbb{E}[Y \mid T = 0, W]\right]$$

Sensitivity Analysis



- Suppose
  - some confounders W are observed
  - others U are unobserved
- If U were observed, we would have

$$\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}_{W,U}\left[\mathbb{E}[Y \mid T=1, W, U] - \mathbb{E}[Y \mid T=0, W, U]\right]$$

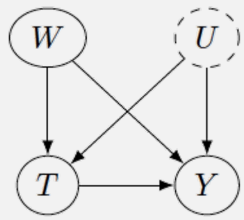
- When U is unobserved, we can only get

$$\mathbb{E}_{W}\left[\mathbb{E}[Y \mid T=1, W] - \mathbb{E}[Y \mid T=0, W]\right]$$

- Can we assess the sensitivity of $\mathbb{E}[Y(1) - Y(0)]$ with respect to U?

Single omitted confounder, linear causal model

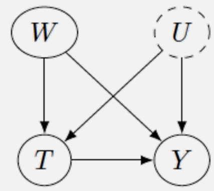
- Goal: recover δ



$$T := \alpha_w W + \alpha_u U$$
$$Y := \beta_w W + \beta_u U + \delta T$$

Single omitted confounder, linear causal model

$$T := \alpha_w W + \alpha_u U$$
$$Y := \beta_w W + \beta_u U + \delta T$$



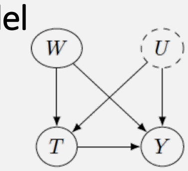
$$\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}_{W,U}\left[\mathbb{E}[Y \mid T=1, W, U] - \mathbb{E}[Y \mid T=0, W, U]\right] = \delta$$

$$\mathbb{E}_W\left[\mathbb{E}[Y \mid T=1, W] - \mathbb{E}[Y \mid T=0, W]\right] \stackrel{?}{=}$$

$$\mathbb{E}_W\left[\mathbb{E}[Y \mid T=1, W] - \mathbb{E}[Y \mid T=0, W]\right] = \delta + \frac{\beta_u}{\alpha_u}$$

$$\text{Bias of } \mathbb{E}_W\left[\mathbb{E}[Y \mid T=1, W] - \mathbb{E}[Y \mid T=0, W]\right] = \delta + \frac{\beta_u}{\alpha_u} - \delta = \frac{\beta_u}{\alpha_u}$$

Single omitted confounder, linear causal model



$T := \alpha_w W + \alpha_u U$

$Y := \beta_w W + \beta_u U + \delta T$

- From the structural equation for Y we have

$$\mathbb{E}_W\left[\mathbb{E}[Y \mid T = t, W]\right] = \mathbb{E}_W\left[\mathbb{E}[\beta_w W + \beta_u U + \delta T \mid T = t, W]\right]$$
$$= \mathbb{E}_W\left[\beta_w W + \beta_u \mathbb{E}[U \mid T = t, W] + \delta t\right]$$

- From the structural equation for T we have

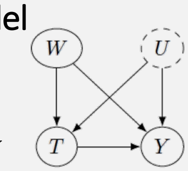
$$U = \frac{T - \alpha_w W}{\alpha_u}$$

- Putting everything together

$$\mathbb{E}_W\left[\mathbb{E}[Y \mid T = t, W]\right] = \mathbb{E}_W\left[\beta_w W + \beta_u\left(\frac{t - \alpha_w W}{\alpha_u}\right) + \delta t\right]$$
$$= \mathbb{E}_W\left[\beta_w W + \frac{\beta_u}{\alpha_u} t - \frac{\beta_u \alpha_w}{\alpha_u} W + \delta t\right]$$
$$= \beta_w \mathbb{E}[W] + \frac{\beta_u}{\alpha_u} t - \frac{\beta_u \alpha_w}{\alpha_u} \mathbb{E}[W] + \delta t$$
$$= \left(\delta + \frac{\beta_u}{\alpha_u}\right) t + \left(\beta_w - \frac{\beta_u \alpha_w}{\alpha_u}\right) \mathbb{E}[W]$$

Single omitted confounder, linear causal model

$$\mathbb{E}_W\left[\mathbb{E}[Y \mid T=t, W]\right] = \left(\delta + \frac{\beta_u}{\alpha_u}\right) t + \left(\beta_w - \frac{\beta_u \alpha_w}{\alpha_u}\right) \mathbb{E}[W]$$



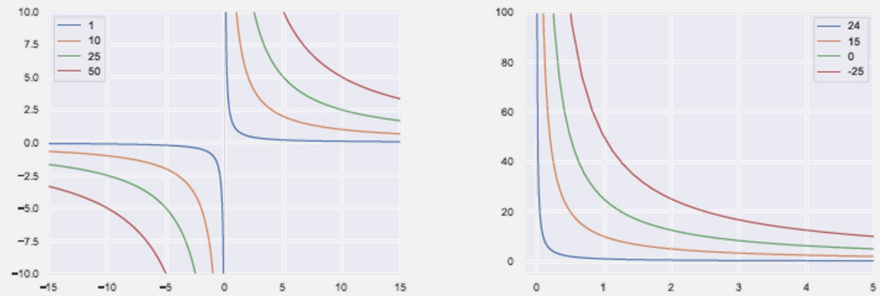
- So the ATE estimate we have if we adjust for only W

$$T := \alpha_w W + \alpha_u U$$
$$Y := \beta_w W + \beta_u U + \delta T$$

$$\begin{aligned}
&\mathbb{E}_W\left[\mathbb{E}[Y \mid T=1, W] - \mathbb{E}[Y \mid T=0, W]\right] \\
&\quad = \left(\delta + \frac{\beta_u}{\alpha_u}\right)(1) + \left(\beta_w - \frac{\beta_u \alpha_w}{\alpha_u}\right) \mathbb{E}[W] \\
&\quad\quad - \left[\left(\delta + \frac{\beta_u}{\alpha_u}\right)(0) + \left(\beta_w - \frac{\beta_u \alpha_w}{\alpha_u}\right) \mathbb{E}[W]\right] \\
&\quad = \delta + \frac{\beta_u}{\alpha_u}
\end{aligned}$$

$$\begin{aligned}
\text{Bias} = &\ \mathbb{E}_W\left[\mathbb{E}[Y \mid T=1, W] - \mathbb{E}[Y \mid T=0, W]\right] \\
&- \mathbb{E}_{W,u}\left[\mathbb{E}[Y \mid T=1, W, U] - \mathbb{E}[Y \mid T=0, W, U]\right] \\
&= \delta + \frac{\beta_u}{\alpha_u} - \delta \\
&= \frac{\beta_u}{\alpha_u}
\end{aligned}$$

Sensitivity contour plots



(a) Contours of confounding bias $\frac{\beta_u}{\alpha_u}$

(b) Contours of the true ATE δ, given that $\mathbb{E}_W\,[\mathbb{E}[Y \mid T = 1, W] - \mathbb{E}[Y \mid T = 0, W]] = 25$

Generalizing sensitivity analysis

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