Center for Artificial Intelligence Foundations & Scientific Applications
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Artificial Intelligence

The Very Idea

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First order logic

Allows us to

- Talk about entire collections of objects or individuals
  - All men are mortal
- Reason from general to the particular
  - Premises:
    - All men are mortal
    - Socrates is a man
  - Conclusion:
    - Socrates is mortal
- Talk about existence of individuals or objects with certain properties
- There exists a student in AI100 that is smart

First Order Predicate Logic

- Propositional logic
  - assumes the world can be represented using propositions
  - has limited expressive power
- First-order predicate logic (like natural language)
  - assumes the world contains
    - Objects:
      - people, flowers, houses, numbers, students,
    - Relations:
      - red, round, prime, brother of, bigger than, part of
    - Functions:
      - father of, best friend, plus, …
  - Allows one to talk about some or all of the objects

Ontological and Epistemological Commitments

	Ontological Commitments	Epistemological Commitments
Propositional Logic	Facts	True, False
First Order Logic	Facts, Objects, Relations, Functions	True, False

FOL Syntax: Basic

- A term is used to denote an object in the world
  - constant: $PennState,\ AI100,\ Joe$
  - variable: $x, y, a, b, c, \ldots$
  - function$(term_1, \ldots, term_n)$:
    - maps one or more objects to a *single object*
    - can be used to refer to an unnamed object: e.g., $Fatherof(John)$
    - cannot be used with logical connectives
- A ground term is a term with no variables
  $Joe, Fatherof(John)$

FOL Syntax: Basic

- An atom is a primitive sentence to which a truth value can be assigned – analogous to atomic proposition in propositional logic
  - predicate($term_1, \ldots, term_n$) is a relation that can be satisfied by multiple instances
    - $Instructor(Vasant, AI100)$
      - True if $Vasant$ is an instructor of $AI100$
    - $Brother(John, James)$
      - True if $John$ is a brother of $James$
    - $Brother(Michael, James)$
      - True if $Michael$ is a brother of $James$
    - Note: $Brother$ is symmetric, $Instructor$ is not
  - $term_1 = term_2$ if both refer to the same object
    - $Houseof(John) = Houseof(Jill)$

FOL Syntax: Basic

- A sentence represents a fact in the world that is assigned a truth value
  - atom
  - compound sentence constructed using connectives
    - $Father(John, James) \land Mother(Jill, James)$
  - Compound sentences constructed using quantified variables
    - $\forall x\ Student(x, AI100) \rightarrow Smart(x)$
      - All students enrolled in $AI100$ are smart
    - $\exists x\ Student(x, AI100) \land Smart(x)$
      - There is a student in $AI100$ that is smart

Semantics: Truth in first-order logic

- Sentences are true with respect to a model and an interpretation
- Model contains relations among objects
- Interpretation specifies referents for
  - constant symbols refer to objects
  - predicate symbols refer to relations
  - function symbols refer to functions

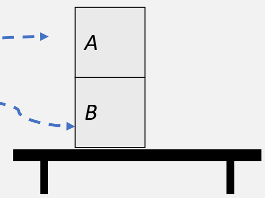
- An atomic sentence $predicate(arg1, arg2\ , argn)$ is true iff the objects referred to by $arg1, arg2\ \ ...\ argn$ are in the relation referred to by $predicate$

Semantics: Truth in first-order logic

- Object Constants: $A, B, Table$
- Relation Constant: On

Model

- $\{On\ (A, B),\quad On\ (B, Table)\}$



Quantifiers

Quantifiers

- Allow us to express properties of collections of objects instead of enumerating objects by name
  - Universal: “for all” $\forall$
  - Existential: “there exists” $\exists$
    - All humans are mortal
    - Every pet dog has a human who cares for it

$$\forall x \;\; Human(x) \Rightarrow Mortal(x)$$

$$\forall z \;\; Petdog(z) \Rightarrow \exists y \; Human(y) \wedge \; Caresfor(y,z)$$

Universal quantification

- ∀<*variables*> <*sentence*>
- Every student in AI100 is smart:
  - $\forall x\ Student(x, AI100) \rightarrow Smart(x)$
- $\forall x\ P(x)$ is true in a model M if and only if P is true for every possible instance of x from the domain of x
- Roughly speaking, $\forall x\ P(x)$ is equivalent to the conjunction of instantiations of P

$(Student(Joe, AI100) \rightarrow Smart(Joe))$

$\wedge(Student(Jill, AI100) \rightarrow Smart(Jill))$

$\wedge(Student(Twinof(Jill), AI100)) \ \rightarrow Smart(Twinof(Jill))$ ….

Existential quantification

$\exists$<*variables*> <*sentence*>

Some student in AI100 PSU is smart:

$$\exists x\, Student(x, AI100) \wedge Smart(x)$$

- $\exists x\, P(x)$ is true in a model *M* iff P is true with x for some instance of x
- Roughly speaking, $\exists x\, P(x)$ equivalent to the disjunction of instantiations of $P\,(x)$

$$(Student(Joe, AI100) \wedge Smart(Joe)) \vee$$
$$(Student(Jane, AI100) \wedge Smart(Jane)) \vee \ldots$$

Common mistake to avoid

- Suppose x is a variable whose domain is the set of all animals
- When we mean
  - $\forall x\ (Human(x) \rightarrow\ Mammal(x))$
  - All humans are mammals
- We incorrectly instead
  - $\forall x\ (Human(x) \land Mammal(x))$
  - Every animal is a human and a mammal

Common mistake to avoid

- When we mean to say
  - There is some human who is president
  - $\exists x\ (Human(x) \wedge President(x))$
- We incorrectly say instead
  - $\exists x\ \big(Human(x) \rightarrow President(x)\big)$
  - Which is true even if no one is human
- For example:
  - $(Human(Socks) \rightarrow President(Socks)) \vee$
    $(Human(Meao) \rightarrow President(Meao)) \vee$
    $(Human(Bowwow) \rightarrow President(Bowwow)) \vee \ \ldots$

FOL Syntax: Quantifiers

- Properties of quantifiers:
  - $\forall x\, \forall y$ is the same as $\forall y\, \forall x$
  - $\exists x\ \ \exists y$ is the same as $\exists y\, \exists x$
  - Note that
    - $\exists x\, \exists y$ can be written as $\exists x, y$
    - $\forall x\, \forall y$ can be written as $\forall x, y$

- Examples
  - $\forall x\, \forall y\, Likes(x, y)$
  - Everyone likes everyone

FOL Syntax: Quantifiers

- Properties of quantifiers:
  - $\forall x\ \exists y$ is **not** the same as $\exists y\ \forall x$
  - $\exists x\ \forall y$ is **not** the same as $\forall y\ \exists x$

- Examples
  - $\boldsymbol{\forall x\ \exists y\ Likes(x, y)}$ Everyone likes someone.
    - $Likes\ (John, Mary)$
    - $Likes\ (Jim, Jill)$
    - $Likes\ (Jill, Mary)$
  - $\exists y\ \forall x\ Likes(x, y)$ Someone is liked by everyone.
    - $Likes\ (John, Mary)$
    - $Likes\ (Jim, Mary)$
    - $Likes\ (Jill, Mary)$ …

FOL Syntax: Quantifiers

- Properties of quantifiers:
  - $\forall x\ P(x)$ is the same as $\neg\exists x\ \neg P(x)$
  - $\exists x\ P(x)$ is the same as $\neg\forall x\ \neg P(x)$

- Examples
  - $\forall x\ Likes(x, IceCream)$
    Everyone likes ice cream.
  - $\neg\exists x\ \neg Likes(x, IceCream)$
    No one doesn't like ice cream.
- Double negative makes a positive ☺

FOL Syntax: Quantifiers

- Properties of quantifiers – De Morgan's laws
  - $\forall x\ P(x)$ when negated is $\exists x\ \neg P(x)$
  - $\exists x\ P(x)$ when negated is $\forall x\ \neg P(x)$

- Examples
  - $\forall x\ Likes(x, IceCream)$
    Everyone likes ice cream.
  - $\exists x\ \neg\ Likes(x, IceCream)$
    Someone doesn't like ice cream.

Fun with First-order logic (FOL)

Express the following English sentences in FOL

- Bob is a fish.
  - What are the objects?

    Bob
  - What are the relations?

    is a fish
  - $Fish(Bob)$ a unary relation or property

Fun with First-order logic (FOL)

Express the following English sentences in FOL

- Deb and Sue are women.
  - We'll be casual about plurals
    - $Woman(Deb)$
    - $Woman(Sue)$
- Deb and Sue are friends.
  - Should we use a function? predicate?
  - Predicate because Deb and Sue can have many friends
    - $Friend(Deb, Sue)$

Fun with First-order logic (FOL)

Express the following English sentences in FOL

- America bought Alaska from Russia.
  - What are the objects?
    - America, Alaska, Russia
  - What are the relations?
    - Bought(buyer, object, seller)
  - $Bought(America, Alaska, Russia)$

Fun with First-order logic (FOL)

Express the following English sentences in FOL

- Jim collects everything.
  - What are the variables?
    - x with domain that includes ``everything''
  - How are they quantified?
    - all, universal
  - $\forall x\ Collects(Jim, x)$
  - $Collects(Jim, Pencil)\ \wedge\ Collects(Jim, Deb)\ \wedge\ \ldots$
- Not quite precise
  - Did we mean to include Jim in ``everything''?
  - Did we want to include $Collects(Jim, Jim)$?

Fun with FOL

When to restrict the domain, e.g., to people:

- All: $\forall x\ (Person(x) \wedge \ldots)\ \ldots$
  - things: anything, everything, whatever
  - people: anybody, anyone, everybody, everyone, whoever
- Some (at least one): $\exists x\ Person(x) \wedge \ldots \wedge \ldots$
  - things: something
  - people: somebody, someone
- None
  - things: nothing: $\neg\exists x\ Person(x) \wedge \ldots \wedge \ldots$
  - people: nobody, no one

Fun with FOL

Express the following English sentences in FOL

- Everybody collects everything.
- Everybody collects something.
- Something is collected by everybody.

Fun with FOL

Express the following English sentences in FOL

- Everybody collects everything.
  $\forall x \forall y\ Person(x) \land Thing(y) \rightarrow Collects(x, y)$
- Everybody collects something.
  $\forall x[Person(x) \land \exists y[Thing(y) \land Collects(x, y)]]$
- Something is collected by everybody.
  $\exists y \forall x Person(x) \land Thing(y) \land Collects(x, y)$

Fun with FOL

Express the following English sentences in FOL

- Somebody collects something.
  - What are the variables?
    - somebody x and something y
  - How are they quantified?
    - at least one, existential
  - Do we want to include persons among collectibles?
  - Should a person be able to collect himself or herself?
    - $\exists x, y\ Person(x) \wedge Collects(x, y)$
  - What if we don't want to include persons among collectibles?
    - $\exists x, y\ Person(x) \wedge Thing(y) \wedge Collects(x, y)$

Fun with FOL

Express the following English sentences in FOL

- All hoarders collect everything.
  - How are ideas connected?
    - being a hoarder implies collecting everything
    - $\forall x, y\ Horder(x) \rightarrow\ Collects(x, y)$
- Hoarders collect valuable things - ambiguous!
  - All hoarders collect all valuable things.
  - All hoarders collect some valuable things.
  - Some hoarders collect all valuable things.
  - Some hoarders collect some valuable things.

Fun with FOL

Express the following English sentences in FOL

- All players who want to play are allowed to play
  - How are ideas connected?
    - Being a player and wanting to play implies being allowed to play

$$\forall x\ (Player(x) \land WantstoPlay(x)) \rightarrow AllowedtoPlay(x)$$

- No player who wants to play is allowed to play
- No player is allowed to play

Fun with FOL

Express the following English sentences in FOL

- Any good amateur can beat some professional.
- Some professionals can beat all amateurs.

Fun with FOL

Express the following English sentences in FOL

- There is exactly one shoe.
  - $\exists x\, Shoe(x) \land \forall y (Shoe(y) \rightarrow (x=y))$

- There are exactly two shoes.
  - Are quantities specified?
  - Are equalities implied?
  - $\exists x,y\, Shoe(x) \land Shoe(y) \land \neg (x=y) \land \forall z\, (Shoe(z) \rightarrow (x=z) \lor (y=z))$

Fun with FOL

- Interesting words: always, sometimes, never
  - Good people always have friends.
    Do we mean ``all good people have friends"?
    $\forall x\ Person(x)\ \wedge\ Good(x)\ \rightarrow\ \exists y(Friend(x,y))$
  - Busy people sometimes have friends.
    May be we mean some busy people have friends?
    $\exists x\ Person(x)\ \wedge\ Busy(x)\ \wedge\ \exists y(Friend(x,y))$
  - Bad people never have friends.

  May be we really mean bad people have no friends.
    ∀x Person(x) ∧ Bad(x) ⇒ ¬∃y(Friend(x,y))
    or equivalently: No bad people have friends.
    ¬∃x Person(x) ∧ Bad(x) ∧ ∃y(Friend(x,y))

Inference with FOL

$\forall x Human(x) \rightarrow Mortal(x)$
$Human(Joe)$

$Mortal(Joe)$

Essentially, we apply sound rules of inference by selectively substitute universally quantified variables with

- objects from their respective domains
- functional expressions that return objects in their domains
- or other variables with identical domains

Inference with FOL

- We can convert FOL sentences into a set of disjunctions with only universally quantified variables
- We can then apply inference rules, e.g., resolution, under truth-preserving substitutions