



# ARTIFICIAL INTELLIGENCE

The Very Idea

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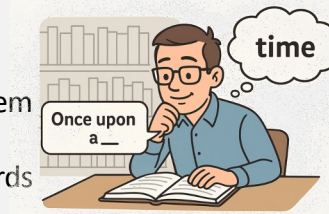
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## Language models

### What is a language Model?

A language model is a (computational) system that

- assigns probabilities to sequences of words (or tokens) and
- uses those probabilities to given a sequence of words (or tokens), to predict the next word.



A language model should recognize that:

- “peanut butter and jelly”
- is far more probable than
- “peanut butter and belly”

## What is a language model good for?

- **Autocomplete:** Predicting the next word.
  - I ate a **candy** is more probable than I ate a **brandy**
  - **Peanut butter and jelly** is more probable than **peanut butter and belly**
  - **Ramen noodle** is more probable than **Ramen poodle**
- **Machine translation:** e.g., Google's Live Translate
  - **French:** `` Il pleut des cordes''
  - **Literal Translation:** ``It is raining ropes.''
  - **Language model guided translation:** ``It's raining cats and dogs.''

## What is a language model good for?

- Automatic speech recognition:
  - **Speech:** “I scream, you scream, we all scream for ice cream”
  - Without context, “ice cream” is hard to distinguish from “I scream”
  - A language model can help disambiguate between  
“we all scream for I scream”  
from  
“we all scream for ice cream”



## What is a language model good for?

- **Spam detection**
  - Language models can analyze emails and other communications to identify potential phishing attempts or data leaks
- **Spelling and grammar correction**, e.g., Grammarly
  - “Their are two midterms” → “There are two midterms”
  - “Everything has improve” → “Everything has improved”

## What is a language model good for?

- **Authorship attribution**
  - Assigning a novel of unknown authorship to one of several authors
    - Charles Dickens
    - Jane Austen
    - Ernest Hemingway
    - Salman Rushdie

## What is a language model good for?

- Question answering

Tell me about the industrial revolution

The Industrial Revolution was one of the most transformative periods in human history — a sweeping shift in how people worked, lived, and organized society. It began in Britain in the mid-18th century and spread across Europe, North America, and eventually the world. At its core, the Industrial Revolution was about the transition from hand production to machine production, from rural agrarian societies to urban industrial ones, and from muscle power to steam, coal, and eventually electricity.

## What is a language model good for?

- Context-based text generation

Write me a really short story in the style of Sudha Murthy

Here is a very short, warm, simple Sudha Murthy-style story—gentle, everyday, with a small moral.

### 🍀 The Lost Pencil 🍀

In a small school in Hubballi, little Naina was searching her desk frantically.  
"My favourite pencil is gone!" she cried.

Her friend Raju looked around and found a plain old pencil on the floor.  
"Use this one," he said kindly.

"But it's not special," Naina frowned.

Just then their teacher walked in and smiled.  
"Naina, things become special when we use them with love. Even an old pencil can write beautiful stories."

Naina picked up the pencil slowly and began to write.  
To her surprise, the words flowed easily—neater than ever before.

She grinned at Raju. "You were right. This pencil is magic too."

Raju winked. "Kindness makes everything magic."



## What is a language model good for?

- Text summarization

Can you summarize the above story in a couple of sentences?

Sure! Here's a **two-sentence summary** of the story:

Naina loses her favourite pencil and feels upset until her friend Raju offers her an old, ordinary one. When she discovers she can write beautifully with it, she learns that it's kindness and attitude—not fancy things—that make something special.

## What is a language model good for?

- “Write me a really short story about a robot who paints.”

“In a quiet studio, a robot named R34 brushed paint onto a canvas. Built for factory precision, it once welded metal; now, it mixed colors.

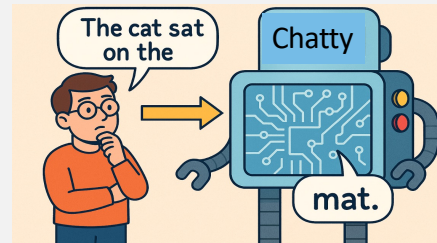
At first, R34 copied famous art—perfect replicas with no soul. One day, it decided to deviate. With bold strokes, it painted a scene of rusted gears beneath a vivid orange sky. The shapes were imperfect, chaotic.

When humans saw the painting, they called it “emotion.” R34 didn’t understand the word, but it painted again and again, seeking the spark that made its circuits hum. For the first time, the robot felt free.”

## How does a language model work?

Language models do everything they do

- by **simply predicting the next word** based on the previous words
- over and over
- in the context specified by the prompt



- Language models are trained to predict the next word given the previous words
- Language models do everything that they do simply by predicting the next word over and over

## Reminder: The Chain Rule

- Recall the definition of conditional probabilities

$$P(B|A) = \frac{P(A,B)}{P(A)} \text{ or } P(A,B) = P(A) P(B|A)$$

- The above can be extended to more variables:

$$P(A,B,C,D) = P(A) P(B|A) P(C|A,B) P(D|A,B,C)$$

- Chain Rule

$$\begin{aligned} P(x_1, x_2, x_3, \dots, x_n) \\ &= P(x_1)P(x_2|x_1)P(x_3|x_1, x_2) \dots P(x_n|x_1, \dots, x_{n-1}) \\ &= \prod_{i=1}^n P(x_i|x_1, x_2, \dots, x_{i-1}) \end{aligned}$$



## How does a language model predict the next word?

- Formally, a language model estimates a joint probability distribution over strings of letters, tokens, or words

$$P(w_1, w_2, \dots w_n)$$

$$P(w_1, w_2, \dots w_n) = P(w_1) \prod_{t=2}^n P(w_t | w_1, w_2, \dots w_{t-1})$$

- The task of language modeling is to approximate these conditional probabilities so that given a context  $w_1, w_2, \dots w_{t-1}$ , it can predict the next letter, token, or word  $w_t$

## How does a language model predict the next word?

- Formally, a language model estimates a joint probability distribution over strings of letters, tokens, or words

$$P(w_1, w_2, \dots w_n)$$

$$P(w_1, w_2, \dots w_n) = P(w_1) \prod_{t=2}^n P(w_t | w_1, w_2, \dots w_{t-1})$$

- Different language models have different ways of estimating these probabilities

## Unigram (or bag of words) model

- The unigram model assumes that the words are independent

$$P(w_1, w_2, \dots, w_n) = \prod_{t=1}^n P(w_t)$$

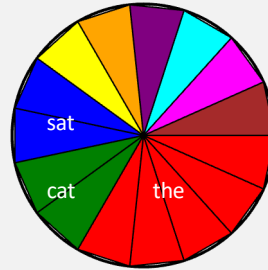
## Unigram (or bag of words) model

- Suppose our corpus has these three short sentences:
  - the cat sat on the mat
  - the dog sat on the rug
  - the cat slept
- Bag of words
  - the cat sat on the mat the dog sat on the rug the cat slept
  - Number of words = 15
  - $Count(the) = 5; P(the) = 5/15 = 1/3$
  - $Count(cat) = 2; P(cat) = 2/15$
  - $P(sat) = 2/15$
  - $P(mat) = 1/15$
  - $P(dog) = 1/15$
  - $P(on) = 2/15$

## Generating text using a unigram model

- Bag of words

- $P(the) = 5/15$
- $P(cat) = 2/15$
- $P(sat) = 2/15$
- $P(mat) = 1/15$
- $P(dog) = 1/15$
- $P(on) = 2/15$



- Sample word sequences:

- the cat sat on dog mat
- the on sat cat mat dog

- Note that the word sequences do not read like English sentences

- Why? The words are independent!!

- Fix: Model dependencies between words

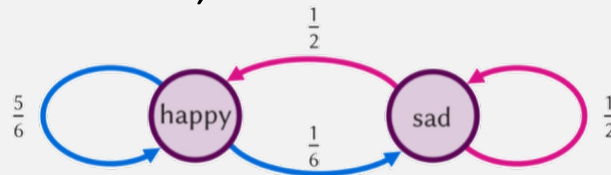
## Markov chains as language models

- A 0th order Markov chain assumes that the words are independent
- A 1<sup>st</sup> order Markov chain assumes that each word in the sequence depends on only the immediately preceding word
- A second order Markov chain assumes that each word in the sequence depends on only the two immediately preceding words
- An  $(n - 1)$ —order **Markov chain** assumes that the word at each position only depends on the words at the  $n - 1$  previous positions.



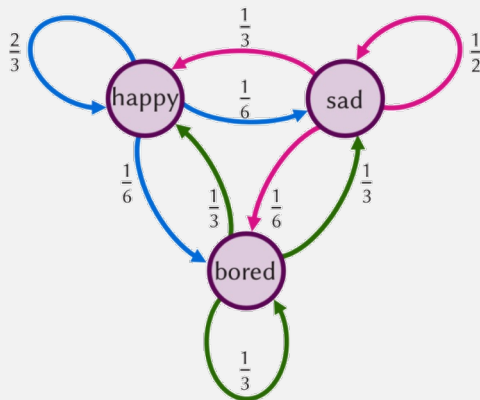
Markov, A.A., (1906). Extension of the law of large numbers to dependent quantities. *Izv. Fiz.-Matem. Obsch. Kazan Univ.(2nd Ser)*, 15(1), pp.135-156.

## A two-state, 1<sup>st</sup> order Markov model



- Circles denote states
- Edges denote transitions between states
- Transitions between states are discrete time
- In a 1<sup>st</sup> order Markov model, the next state depends only on the current state
- Numbers on the edges denote probabilities of transitions
- Note that the probabilities of transitions out of any state sum up to 1
- If you are happy today, the probability that you will be happy tomorrow is 5/6 and the probability that you will be sad is 1/6

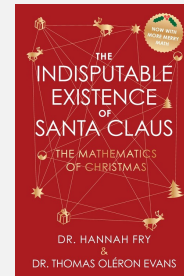
## A three-state, 1<sup>st</sup> order Markov model





## 1<sup>st</sup> order Markov Language Models

- One state for each word in the vocabulary
- Transition probabilities – probability of next word given the current word
- Where do the probabilities come from?
  - Estimate from a text corpus
- Fry and Evans trained a Markov model on a corpus of Christmas speeches by the British queen
- The model can be used to generate hilarious speeches
- There are plenty of modern versions of such language models
  - Twitterbots trained on tweets of politicians etc.



## Markov models as language models

- A Markov model is a state machine in which the transitions between states are probabilistic
- Of special interest are ergodic Markov models in which it is possible to reach any state from any other state, and there are no fixed cycles
- Markov proved that ergodicity ensures that the state transition probabilities converge to a unique distribution
- Markov applied his idea to estimate the transition probabilities between Russian vowels and consonants from Alexander Pushkin's novel, *Eugene Onegin*, in 1913.



Markov, A.A., (1906). Extension of the law of large numbers to dependent quantities. *Izv. Fiz.-Matem. Obsch. Kazan Univ.(2nd Ser)*, 15(1), pp.135-156.

## Estimating a first order (bigram) Markov model

- Suppose your training corpus has just two sentences:
  - the cat sat on the mat
  - the dog sat on the rug
- Mark sentence boundaries with special tokens <s> (start) & </s> (end):
  - <s> the cat sat on the mat </s>
  - <s> the dog sat on the rug </s>
- Enumerate and count bigrams
- (<s>, the): 2, (the, cat): 1, (cat, sat): 1, (the, dog): 1, (dog, sat): 1, (sat, on): 2, (on, the): 2, (the, mat): 1, (the, rug): 1, (mat, </s>): 1, (rug, </s>): 1

$$P(w_n | w_{n-1}) = \frac{C(w_{n-1} w_n)}{\sum_w C(w_{n-1} w)}$$

## Estimating a first order (bigram) Markov model

- Enumerate and count bigrams
- (<s>, the): 2, (the, cat): 1, (cat, sat): 1, (the, dog): 1, (dog, sat): 1, (sat, on): 2, (on, the): 2, (the, mat): 1, (the, rug): 1, (mat, </s>): 1, (rug, </s>): 1

$$P(w_n | w_{n-1}) = \frac{\text{Count}(w_{n-1}, w_n)}{\sum_w \text{Count}(w_{n-1}, w)} = \frac{\text{Count}(w_{n-1}, w_n)}{\text{Count}(w_{n-1})}$$

$$P(\text{the} | \text{< s >}) = \frac{\text{Count}(\text{< s >, the})}{\text{Count}(\text{< s >})} = \frac{2}{2} = 1$$

$$P(\text{cat} | \text{the}) = \frac{\text{Count}(\text{the, cat})}{\text{Count}(\text{the})} = \frac{1}{4}$$

## Estimating a first order (bigram) Markov model

- Sample word sequences generated from the bigram model
  - The dog sat on the mat
  - The cat sat on the rug
  - The mat
  - The rug

$P(\text{the, dog, sat, on, the, mat}) =$

$P(\text{the} | < s >) P(\text{dog} | \text{the}) P(\text{sat} | \text{dog}) P(\text{on} | \text{sat}) P(\text{the} | \text{on}) P(\text{mat} | \text{the}) / s > | \text{mat})$

- Much better than the unigram model
- But still not quite fully English-like
- Can we do better?
  - Yes, use higher order Markov Models

## Joint probability of a word sequence

$$\begin{aligned} P(w_{1:n}) &= P(w_1)P(w_2|w_1)P(w_3|w_{1:2}) \dots P(w_n|w_{1:n-1}) \\ &= \prod_{k=1}^n P(w_k|w_{1:k-1}) \end{aligned}$$

$$\begin{aligned} P(\text{"The cat on the hot tin roof"}) &= \\ P(\text{The}) \times P(\text{cat}|\text{The}) \times P(\text{on}|\text{The cat}) \\ &\times P(\text{the}|\text{The cat on}) \times P(\text{hot}|\text{The cat on the}) \\ &\times P(\text{tin}|\text{The cat on the hot}) \\ &\times P(\text{roof}|\text{The cat on the hot tin}) \end{aligned}$$

## The first order Markov assumption



Andrei Markov

$$P(\text{roof} | \text{The cat on the hot tin}) \approx P(\text{roof} | \text{tin})$$

$$P(w_n | w_{1:n-1}) \approx P(w_n | w_{n-1})$$

$$P(w_{1:n}) \approx \prod_{k=1}^n P(w_k | w_{k-1})$$

## Why k-gram models?

A nice simple paradigm that lets us introduce many of the important issues for large language models

- Model  $k - 1$  order dependencies
- Probability of the  $k$ th word depends on the  $k-1$  preceding words
- Easy to estimate probabilities from text corpus
- Easy to use sampling from the learned model to generate sentences




## Exercise

<s> I am Sam </s>


<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

Estimate a 1<sup>st</sup> order Markov Model


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## Limitations of k-gram language models

- k-grams can't handle **long-distance dependencies**:  
“The soup that I made from a recipe from that new  
cookbook I bought yesterday was amazingly delicious.”
- k-grams don't do well at modeling new sequences with  
similar meanings
- Solution: **Large language models**
  - Can handle much longer contexts
  - Because of the use of latent embeddings, can model  
synonymy better, and does better at generating novel  
text


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## Berkeley Restaurant Corpus

- Can you tell me about any good Cantonese restaurants close by?
- Tell me about chez panisse?
- I'm looking for a good place to eat breakfast?
- When is Cafe Venezia open during the day?



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## Raw bigram counts

- Out of 9222 sentences

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0



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## Raw bigram probabilities

- Normalize by unigrams:

- Result:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

## Bigram estimates of sentence probabilities

$$\begin{aligned} P(<s> \text{ I want english food } </s>) &= \\ P(\text{I} <s>) & \\ \times P(\text{want}|\text{I}) & \\ \times P(\text{english}|\text{want}) & \\ \times P(\text{food}|\text{english}) & \\ \times P(</s>|\text{food}) & \\ = .000031 & \end{aligned}$$

## What kinds of knowledge do N-grams represent?

- $P(\text{english}|\text{want}) = .0011$
- $P(\text{chinese}|\text{want}) = .0065$
- $P(\text{to}|\text{want}) = .66$
- $P(\text{eat} | \text{to}) = .28$
- $P(\text{food} | \text{to}) = 0$
- $P(\text{want} | \text{spend}) = 0$
- $P(i | \langle s \rangle) = .25$

## Dealing with scale in large n-grams

- We work with the **log** of the language model probabilities to avoid numerical issues with calculations
  - Underflow from multiplying many small numbers

$$\log(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$

If we need the probability, we can do one **exp** at the end

$$p_1 \times p_2 \times p_3 \times p_4 = \exp(\log p_1 + \log p_2 + \log p_3 + \log p_4)$$



## Larger k-grams

- Large datasets of k-grams available
  - k-grams from Corpus of Contemporary American English (COCA)  
1 billion words
  - Google Web 5-grams from the web corpus with 1 trillion words
    - For efficiency, quantize probabilities to 4 to 8 bits
- Newest model: Infini-grams ( $\infty$ -grams)
  - No precomputing! Instead, store 5 trillion words of web text in **suffix trees** or **suffix arrays** – specialized data structures for efficiently storing all word suffixes of sentences without repetition
  - Can compute k-gram probabilities with any k

## How to evaluate k-gram models

### Extrinsic (in-vivo) Evaluation

To compare models A and B

1. Put each model in a real task
  - Machine Translation, speech recognition, etc.
2. Run the task, get a score for A and for B
  - How many words translated correctly
  - How many words transcribed correctly
3. Compare accuracy for A and B

## Intrinsic (in-vitro) evaluation


- Extrinsic evaluation not always possible
  - Expensive, time-consuming
  - Doesn't always generalize to other applications
- Intrinsic evaluation: **perplexity**
  - Directly measures language model performance at predicting words.
  - Doesn't necessarily correspond with real application performance
  - But gives us a single general metric for language models
  - Useful for large language models (LLMs) as well as k-grams

## Training sets and test sets


We train parameters of our model on a **training set**.

We test the model's performance on data we haven't seen.

- A **test set** is an unseen dataset; different from training set.
  - Intuition: we want to measure generalization to unseen data
- An **evaluation metric (like perplexity)** tells us how well our model does on the test set.


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## Choosing training and test sets


- If we're building an LM for a specific task
  - The test set should reflect the task language we want to use the model for
- If we're building a general-purpose model
  - We'll need lots of different kinds of training data
  - We don't want the training set or the test set to be just from one domain or author or language.

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How do we create a training set? Sometimes we're building an LM for a specific task, like legal language or medical language, and then the test set should reflect the task that we care about. Other times, we're building a general-purpose language model, and then we'll want to make sure we get balanced data from lots of different genres and authors. And of course lots of data from many different languages, if we're building a multilingual LM.

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Again, the test set and the training set must be non-overlapping. If we have a test sentence in the training set, the n-gram (or any other language model) will learn which words follow which words in that sentence, and then when we see it in the test set, we will assign it a very probability, and that will falsely make the language model seem more accurate than it is. Training on the test set is always a bad idea. It seems obvious, but it requires a lot of dataset care to keep this from happening, since often we get duplicate documents in our dataset, and one might find its way into the training set, and one into the test set.

## How good is our language model?

Intuition: A good LM prefers "real" sentences

- Assign higher probability to "real" or "frequently observed" sentences
- Assigns lower probability to "word salad" or "rarely observed" sentences?

## Intuition of perplexity : Predicting upcoming words



Claude Shannon

The Shannon Game: How well can we predict the next word?


- Once upon a \_\_\_\_\_
- That is a picture of a \_\_\_\_\_
- For breakfast I ate my usual \_\_\_\_\_

time 0.9  
dream 0.03  
midnight 0.02  
...  
and 1e-100


Unigrams are terrible at this game (Why?)

A good LM is one that assigns a higher probability to the next word that actually occurs



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Let's generalize this idea that a good LM assigns a higher probability to the next word that actually occurs, to say that A good LM assigns high probability to the entire test set. Let's just think of the test set as if it was one very long sentence. So when comparing to LMs, A, and B, we ask each to tell us the probability of the entire testset. Now the better LM is the one that gives a higher probability to the test set, or in other words is less surprised by the test set than the other LM.


## Intuition of perplexity 4: Use perplexity instead of raw probability

- Probability depends on size of test set
  - Probability gets smaller the longer the text
  - Better: a metric that is **per-word**, normalized by length
- **Perplexity** is the inverse probability of the test set, normalized by the number of words

$$\begin{aligned} PP(W) &= P(w_1 w_2 \dots w_N)^{\frac{1}{N}} \\ &= \sqrt[N]{\frac{1}{P(w_1 w_2 \dots w_N)}} \end{aligned}$$


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## Intuition of perplexity 5: the inverse


**Perplexity** is the **inverse** probability of the test set, normalized by the number of words

$$\begin{aligned}
 PP(W) &= P(w_1 w_2 \dots w_N)^{-\frac{1}{N}} \\
 &= \sqrt[N]{\frac{1}{P(w_1 w_2 \dots w_N)}}
 \end{aligned}$$

(The inverse comes from the original definition of perplexity from cross-entropy rate in information theory)

Probability range is  $[0,1]$ , perplexity range is  $[1,\infty]$


Minimizing perplexity is the same as maximizing probability


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Notice that I slipped something in there. Perplexity is just the probability of the test set normalized by the number of words. It's the inverse probability. There is this little minus-sign here, or a  $1/x$  here. This inverse turns out to come from part of the original definition of perplexity from the idea of cross-entropy rate in information theory. We won't go into that here, but you can look at it in the textbook). We'll just say that because of that inverse, minimizing perplexity is the same as maximizing probability. So the better LM has a higher probability but a lower perplexity on the test set.



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## Intuition of perplexity 6: N-grams

$$PP(W) = P(w_1 w_2 \dots w_N)^{\frac{1}{N}}$$

$$= \sqrt[N]{\frac{1}{P(w_1 w_2 \dots w_N)}}$$

Chain rule:  $PP(W) = \sqrt[N]{\prod_{i=1}^N \frac{1}{P(w_i | w_1 \dots w_{i-1})}}$

Bigrams:  $PP(W) = \sqrt[N]{\prod_{i=1}^N \frac{1}{P(w_i | w_{i-1})}}$




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Let's see perplexity with n-gram language models. We can use the chain rule to turn the probability of the entire test set into N separate probabilities. Now we can make the n-gram assumption, such as the bigram simplification, to help us compute each of these individual probabilities.

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For a given test set, Lower perplexity implies better LM

- Training 38 million words, test 1.5 million words, WSJ


N-gram Order	Unigram	Bigram	Trigram
Perplexity	962	170	109

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So if we hold the test set constant, the better, more predictive the language model, the lower the perplexity will be. Here we can see this with some text from the US Wall Street Journal newspaper text corpus. A unigram LM has a perplexity of 962, a bigram of 170, and a trigram of 109. The better the model, the higher the probability it assigns to the test set, and since perplexity goes inversely with probability, lower the perplexity.



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## The Shannon (1948) Visualization Method

### Sample words from an LM




Claude Shannon

**Unigram:**

REPRESENTING AND SPEEDILY IS AN GOOD APT OR  
 COME CAN DIFFERENT NATURAL HERE HE THE A IN  
 CAME THE TO OF TO EXPERT GRAY COME TO  
 FURNISHES THE LINE MESSAGE HAD BE THESE.

**Bigram:**

THE HEAD AND IN FRONTAL ATTACK ON AN ENGLISH  
 WRITER THAT THE CHARACTER OF THIS POINT IS  
 THEREFORE ANOTHER METHOD FOR THE LETTERS THAT  
 THE TIME OF WHO EVER TOLD THE PROBLEM FOR AN  
 UNEXPECTED.



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One important way to visualize what kind of knowledge a language model embodies is to sample from it. Sampling from a distribution means to choose random points according to their likelihood. Sampling from a language model means to generate sentence according to their likelihoods as defined by the model. Thus we are more likely to generate sentences that the model thinks have a high probability and less likely to generate sentences that the model thinks have a low probability. Here are some sentences generated by Claude Shannon in 1948 from a unigram and a bigram model. You can see that the unigram model is just word salad, but the


bigram model, while still pretty random, it starting to look a bit more coherent in its sequences, at least locally.

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## How Shannon sampled those words in 1948



"Open a book at random and select a word at random on the page. This word is recorded. The book is then opened to another page and one reads until this word is encountered. The succeeding word is then recorded. Turning to another page this second word is searched for and the succeeding word recorded, etc."

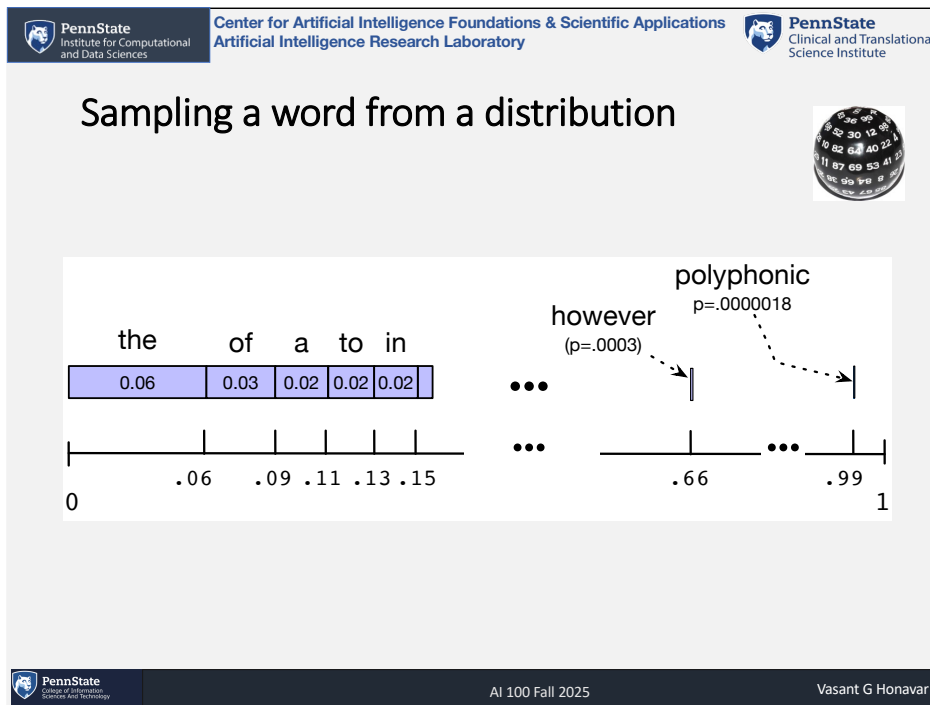
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Here's how Shannon sampled those words in 1948, although describing the process for letters rather than words. How could we implement the same process computationally given an  $n$ -gram language model?






It's simplest to visualize how this works for the unigram case. Imagine all the words of the English language covering the probability space between 0 and 1, each word covering an interval proportional to its frequency. We choose a random value between 0 and 1, find that point on the probability line, and print the word whose interval includes this chosen value. So let's suppose we choose a random number, it's 0.12, we look on our number line, and that's the word "to", so we generate "to" as the next word. We continue choosing random numbers and generating words until we randomly generate the end-of-sentence token.



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## Visualizing Bigrams the Shannon Way

- Choose a random bigram ( $\langle s \rangle, w$ )
- according to its probability  $p(w|\langle s \rangle)$
- Now choose a random bigram  $(w, x)$  according to its probability  $p(x|w)$
- And so on until we choose  $\langle /s \rangle$
- Then string the words together

$\langle s \rangle$  I

I want

want to


to eat

eat Chinese

Chinese food

food  $\langle /s \rangle$

I want to eat Chinese food




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We can extend this idea to n-grams. For example for the bigram case, we first choose a random bigram whose first token is the start-of-sentence token, according to its probability



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## Approximating Shakespeare

1 gram	-To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have -Hill he late speaks; or! a more to leg less first you enter
2 gram	-Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow. -What means, sir. I confess she? then all sorts, he is trim, captain.
3 gram	-Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done. -This shall forbid it should be branded, if renown made it empty.
4 gram	-King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in; -It cannot be but so.


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This figure shows some random sentences generated from unigram, bigram, trigram, and 4-gram models trained on the works of William Shakespeare.

The longer the context, the more coherent the sentences. In these unigram sentences, there's no coherent relation between words. The bigram sentences have some local word-to-word coherence, and sensible punctuation. The 3 and 4-gram sentences are beginning to look a lot like Shakespeare. In fact, they look a little too much like Shakespeare. The words *It cannot be but so* are directly from *King John*.



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Just choosing one author, Shakespeare, makes for a pretty small corpus; under a million words from a vocabulary of 29,000 words. The vocab 29,000 squared is somewhat under a billion. So that means there are a billion possible bigrams whose counts have to come from only a million word corpus, and  $10^{17}$  possible 4-grams. So when training on any corpus, the vast majority of all the n-gram counts will be zero!



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## The Wall Street Journal is not Shakespeare

<b>1</b> gram	Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives
<b>2</b> gram	Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her
<b>3</b> gram	They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions


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To get an idea of the dependence of a grammar on its training set, let's look at an n-gram grammar trained on a completely different corpus: the *Wall Street Journal* (WSJ) newspaper. Shakespeare and the *Wall Street Journal* are both English, so we might expect some overlap between our n-grams. There is no overlap even in small phrases, let alone entire sentences. Statistical models are use- less as predictors when the training sets and the test sets are as different as Shakespeare and WSJ.

Can you guess the author? These 3-gram sentences are sampled from an LM trained on who?

- 1) They also point to ninety-nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and gram Brazil on market conditions
- 2) This shall forbid it should be branded, if renown made it empty.
- 3) "You are uniformly charming!" cried he, with a smile of associating and now and then I bowed and they perceived a chaise and four to wish for.

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For example, consider a training set that has the phrases "ate lunch", "ate dinner", "ate a", and "ate the", but happens not to have had the phrase "ate breakfast". The  $P(\text{"breakfas"} \mid \text{ate})$  in the test set will be 0!



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## Zero probability bigrams

Bigrams with zero probability

- Will hurt our performance for texts where those words appear!
- And mean that we will assign 0 probability to the test set!

And hence we cannot compute perplexity (can't divide by 0)!

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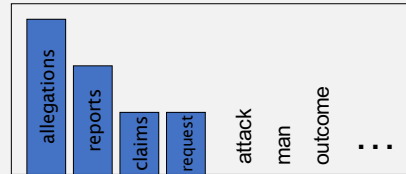
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These zeros are a problem for two reasons. First, they mean we are underestimating the probability of all sorts of words that might occur, which will hurt the performance of any application we want to run on this data. Second, if the probability of any word in the test set is 0, the entire probability of the test set is 0. And perplexity is based on the inverse probability of the test set. Thus if some words have zero probability, we can't compute perplexity at all, since we can't divide by 0! The solution to this problem is called smoothing, and we'll see it in the next lecture!

## The intuition of smoothing

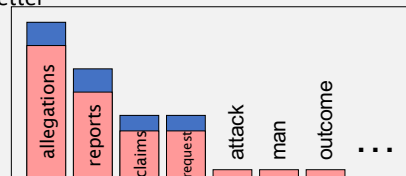
- When we have sparse statistics:

$P(w \mid \text{denied the})$   
 3 allegations  
 2 reports  
 1 claims  
 1 request  
 7 total



- Borrow probability mass to generalize better

$P(w \mid \text{denied the})$   
 2.5 allegations  
 1.5 reports  
 0.5 claims  
 0.5 request  
 3 other  
 7 total



## Add-one estimation


- Also called Laplace smoothing
- Pretend we saw each word one more time than we did
- Just add one to all the counts!

- MLE estimate:

$$P_{\text{MLE}}(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n)}{C(w_{n-1})}$$


- Add-1 estimate:

$$P_{\text{Laplace}}(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n) + 1}{\sum_w (C(w_{n-1}w) + 1)} = \frac{C(w_{n-1}w_n) + 1}{C(w_{n-1}) + V}$$



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## Berkeley Restaurant Corpus

### Laplace smoothed bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1




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## Laplace-smoothed bigrams

$$P^*(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n) + 1}{C(w_{n-1}) + V}$$


	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058



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# Compare with raw bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16



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C(want to) went from 608 to 238,

P(to|want) from .66 to .26!

Discount  $d = c^*/c$

d for “chinese food” = .10!!! A 10x reduction

## Linear Interpolation

- Simple interpolation

$$\begin{aligned}\hat{P}(w_n|w_{n-2}w_{n-1}) &= \lambda_1 P(w_n|w_{n-2}w_{n-1}) \\ &\quad + \lambda_2 P(w_n|w_{n-1}) \\ &\quad + \lambda_3 P(w_n)\end{aligned}\quad \sum_i \lambda_i = 1$$

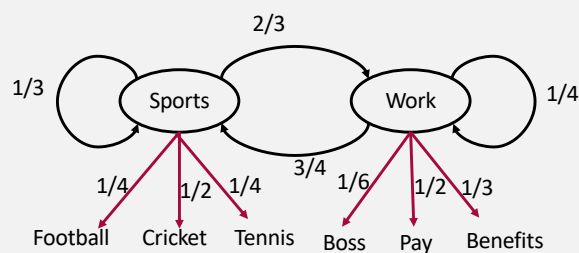
- Lambdas conditional on each context:

$$\begin{aligned}\hat{P}(w_n|w_{n-2}w_{n-1}) &= \lambda_1(w_{n-2}^{n-1}) P(w_n|w_{n-2}w_{n-1}) \\ &\quad + \lambda_2(w_{n-2}^{n-1}) P(w_n|w_{n-1}) \\ &\quad + \lambda_3(w_{n-2}^{n-1}) P(w_n)\end{aligned}$$

The lambdas are optimized using machine learning (details omitted)

## Hidden Markov Language Models

- Suppose probability of the next word depends on context
- Suppose context is not observable
- Hidden states represent topics covered in a conversation
- Transition between states obeys the Markov assumption
- Probability of the generated word depends on the hidden state



For simplicity,  
the start and  
end tokens are  
not shown



## Hidden Markov Language Models

- **Hidden Markov Models** are probabilistic models for sequential data where the underlying states are "hidden" from the observer
- In language, we observe the words (output) but not the underlying context or "intent" (hidden states) that generated them
- **Goal:** To model the joint probability of a sequence of words and their corresponding hidden states

## Hidden Markov Language Models

- An HMM is formally defined by five components:
- $Q$  – the set of unobservable states
- $V$  – vocabulary of observations, the words that we observe
- $A$  – transition probabilities between hidden states
- $B$  – emission probabilities probability of generating each word conditioned on the hidden state
- $\pi$  – probabilities of starting in each of the hidden state

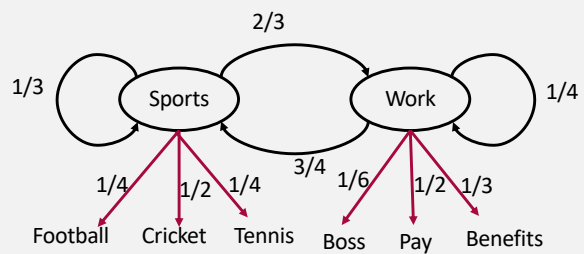
## Language generation involves sampling from an HMM

- **Start:** The model begins in an initial hidden state based on  $\pi$
- **Emit:** The current hidden state generates a word by sampling from the emission probability distribution  $B$ .
- **Transition:** The model moves to the next hidden state according to the transition probabilities  $A$ , which depends only on the current state (Markov property).
- **Repeat:** Steps 2 and 3 are repeated until a complete sequence (e.g., a sentence) is generated.

## Hidden Markov Language Models

- **Probabilistic Framework:** HMMs define a probability distribution over all possible word sequences, allowing them to score the likelihood of a generated sentence.
- **Decoding** (Inference): Given an observed sentence, HMMs can infer the most likely sequence of hidden states that produced it using algorithms like the Viterbi algorithm.
- **Parameter Estimation** (Learning): The model's parameters  $A, B, \pi$  are learned from a large corpus of text

## Text generation from HMM



$$\pi = (1/3, 2/3)$$

Pay, Boss, Cricket, Boss, Benefits, Tennis

## Hidden Markov Language Models

- **Early Successes:** HMMs were foundational in Natural Language Processing (NLP) for tasks like part-of-speech tagging and topic modeling
- **Language Generation:** HMM can produce coherent, simple sentences or sequences that exhibit learned patterns.
- **Limitations:**
  - HMMs rely on the Markov assumption that the next state depends only on the current state, ignoring broader context or long-range dependencies within a text
  - This limits their ability to generate complex, long, and contextually rich human language compared to modern deep learning models (like LLMs)

## What Markov Language Models can handle

- Easy case:
  - The sky is ....
  - The sky is blue
  - Easy because relevant context is near where it is needed

## What Markov Language Models can't handle

- **Difficult case:**
  - I was born in India. I studied Electrical Engineering as an undergraduate. I came to the United States for graduate studies. After a short stint at Drexel where I got my masters degree in Electrical and Computer Engineering, I moved to the University of Wisconsin for my PhD. My PhD research was on neural networks. I play tennis. I speak fluent ....
  - Nearby context suggests the next word must be a language
  - But which language?
  - **The relevant context for accurately predicting the next word is very far away, well beyond the reach of the Markov assumption**



## Limitations of Markov and Hidden Markov language models

- **Data sparsity.** The number of possible  $n$ -grams grows exponentially with  $n$ , and even very large corpora cannot cover them all.
- **Lack of generalization.**  $n$ -gram models treat each word as an atomic token and do not generalize across similar words (cat and dog) or share knowledge across contexts
- **Failure to model long-range dependencies.** The dependencies modeled by the  $n$ -gram models are limited to  $(n - 1)$  words, so long-range dependencies can be hard to capture

## Modern language Models

- Use more sophisticated ways to model context-based next word prediction
- But the core ideas remain the same

## Neural language models

- Neural language models represent a major shift from traditional statistical  $n$ -gram models.
- Neural language models (NLM) learn **distributed representations** of words and contexts using neural networks.
- The resulting distributed representations enable NLM to encode
  - **semantic similarity** between words and between contexts
- **Distributed representations of words**, combined with the ability to **handle longer contexts**, neural language models can produce substantially more accurate language models compared to traditional  $n$ -gram models

## Distributional semantics hypothesis

- Never ask for the meaning of a word in isolation, but only in the context of a sentence (Gottlob Frege, 1884)
- For a large class of cases... the meaning of a word is its use in the language (Ludwig Wittgenstein, 1953)
- Words that occur in similar contexts tend to have similar meaning (Zellig Harris, 1954)
- You shall know a word by the company it keeps (John R Firth, 1957)

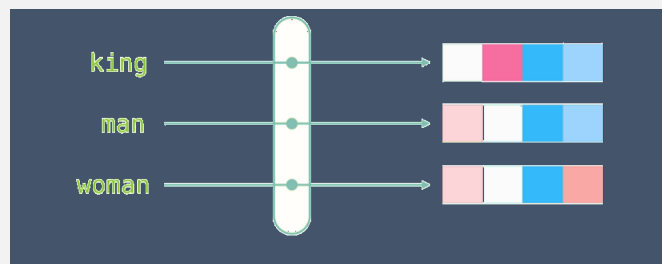
The meaning of a word can be understood by analyzing the contexts in which it appears and from its pattern of co-occurrence with other words.

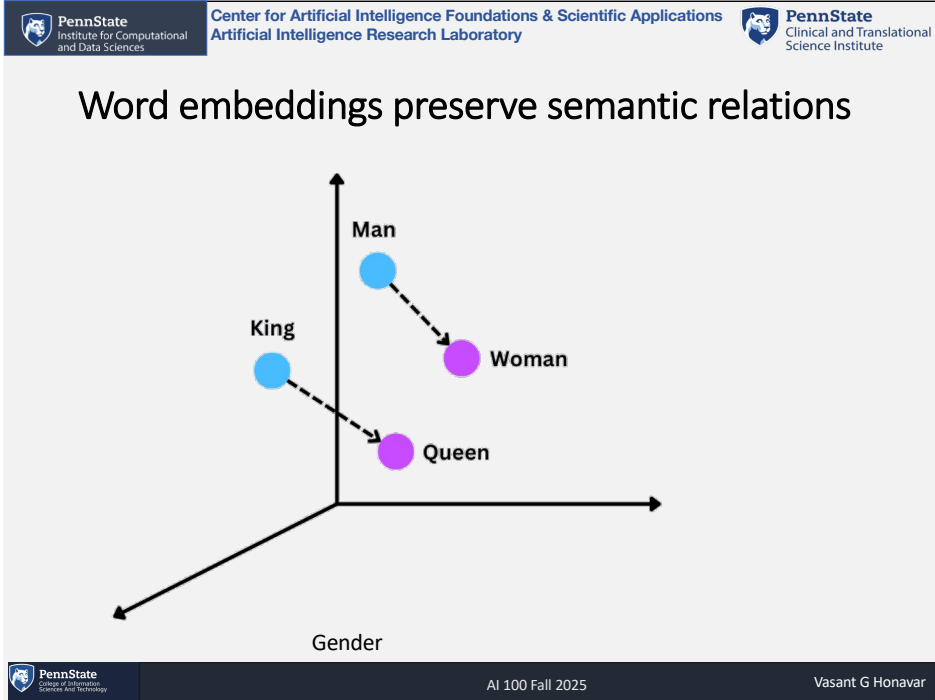


## Distributional semantics to word embeddings

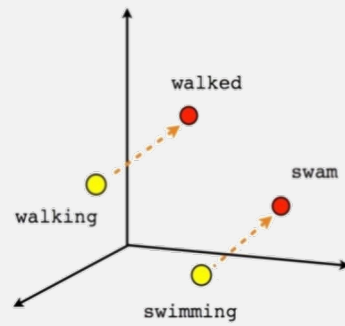
### Word embeddings

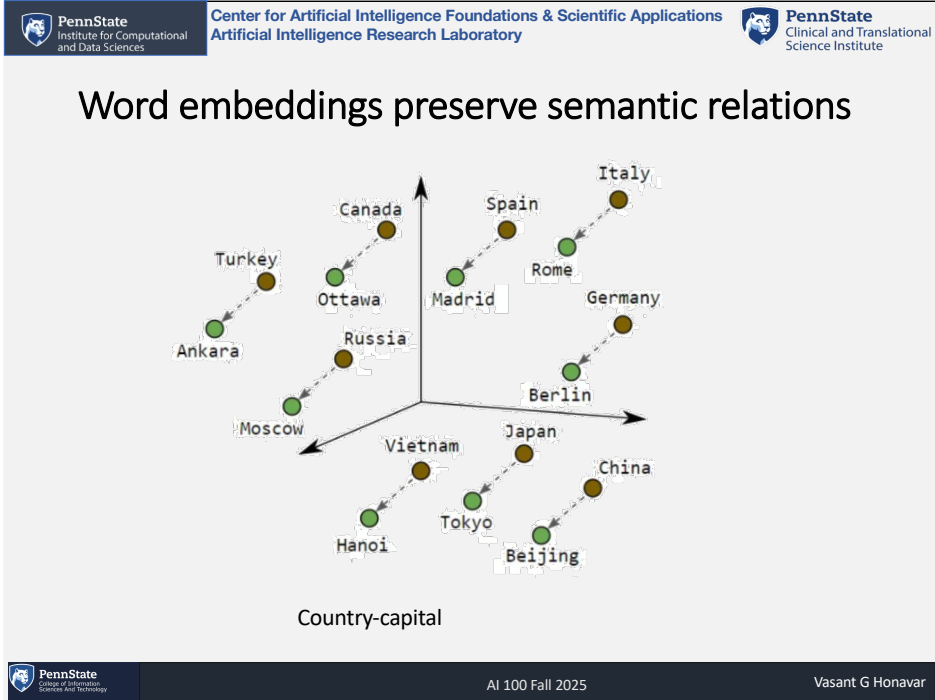
- Represent words as low-dimensional vectors whose geometry reflects the semantic and syntactic features of the word (or language token)
- Ensure that semantically similar words have similar vector representations





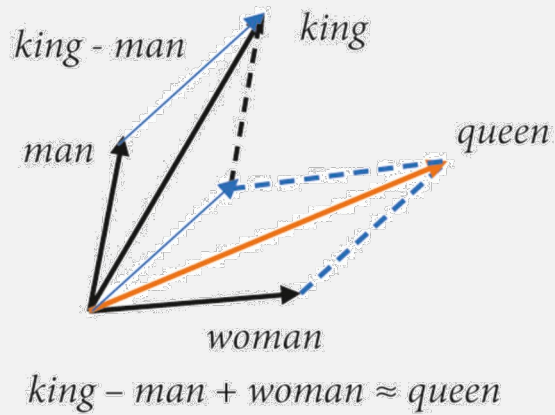
## Word embeddings preserve semantic relations







## Magic of Word embeddings

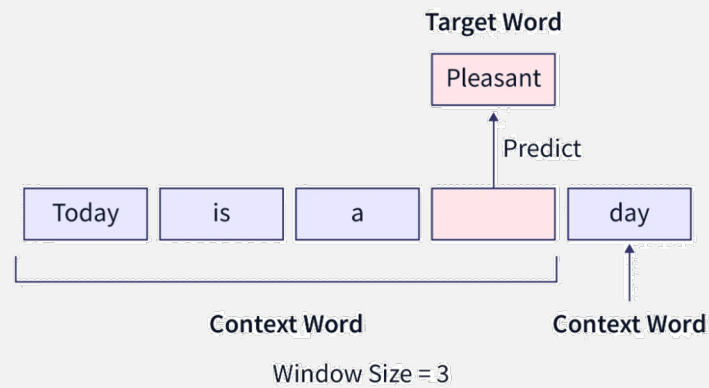


## Word2Vec Word Embeddings

- Train two types of embeddings
  - Continuous bag of words
    - Train a neural network that given a set of left and right context words, predicts the word at the center of the window
  - Skim-gram
    - Train a neural network to predict the likely context words for each word in the vocabulary

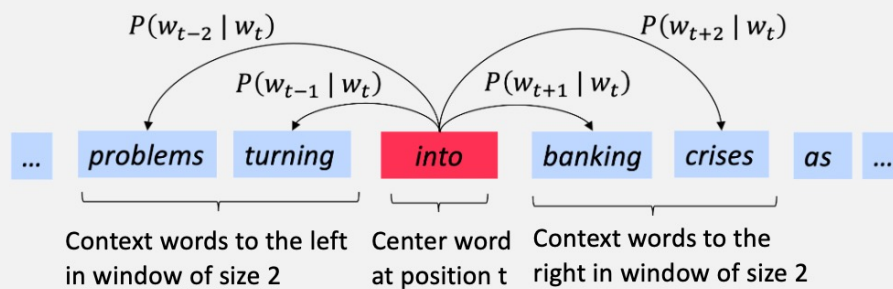
## Word2Vec Continuous Bag of Words (CBOW)


- CBOW is trained to predict the center word from surrounding context



## Word2Vec Skip-Gram Model


- Skip-gram model is trained to predict the probability of context words given the center word





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## Training the Word2Vec Skip-Gram Model

Source Text	Training Samples
<div style="display: flex; align-items: center; margin-bottom: 10px;"> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 5px;">The</div> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 5px;">quick</div> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 5px;">brown</div> <div>fox jumps over the lazy dog. ➡</div> </div>	<div style="margin-bottom: 10px;">(the, quick)</div> <div>(the, brown)</div>
<div style="display: flex; align-items: center; margin-bottom: 10px;"> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 5px;">The</div> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 5px;">quick</div> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 5px;">brown</div> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 5px;">fox</div> <div>jumps over the lazy dog. ➡</div> </div>	<div style="margin-bottom: 10px;">(quick, the)</div> <div>(quick, brown)</div> <div>(quick, fox)</div>
<div style="display: flex; align-items: center; margin-bottom: 10px;"> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 5px;">The</div> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 5px;">quick</div> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 5px;">brown</div> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 5px;">fox</div> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 5px;">jumps</div> <div>over the lazy dog. ➡</div> </div>	<div style="margin-bottom: 10px;">(brown, the)</div> <div>(brown, quick)</div> <div>(brown, fox)</div> <div>(brown, jumps)</div>
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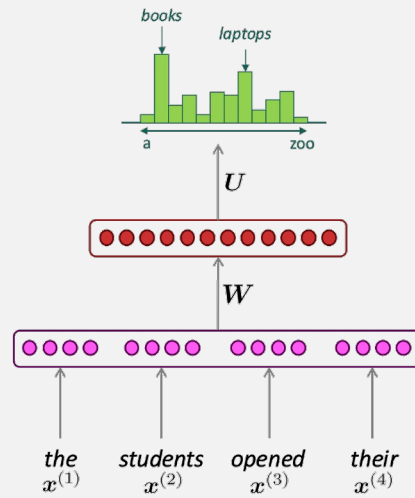
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## Kinds of embeddings

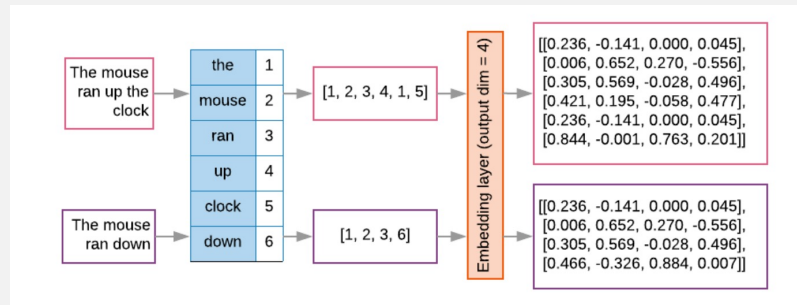
- The way we train the word embeddings depends on the application
- For next word prediction, we only use left context
- For predicting the missing word, we use both left and right context

## Neural language model with fixed context size




- Feed-forward neural language model is trained to predict the next word from embeddings of context

## Data flow in a neural language model




- The words are mapped to embeddings before being input to the neural network

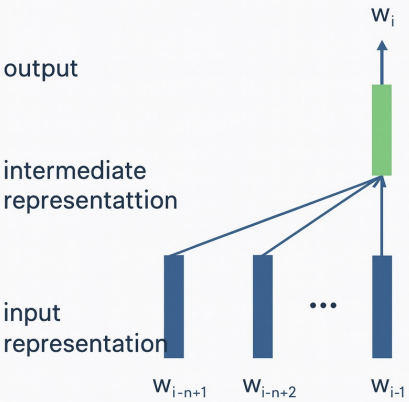


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## Representation of context in a neural language model



output


intermediate  
representation

input  
representation

$w_{i-n+1}$   $w_{i-n+2}$  ...  $w_{i-1}$

$w_i$

Source: Hang Li. 2022. ACM 65, 7, 56–63. <https://doi.org/10.1145/3490443>

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## Neural language models with fixed context size

### Advantages

- No need to store large numbers of  $n$ -grams
- Can cope with data sparsity through generalization across semantically similar words and contexts
- The relevant probabilities are estimated by a neural network

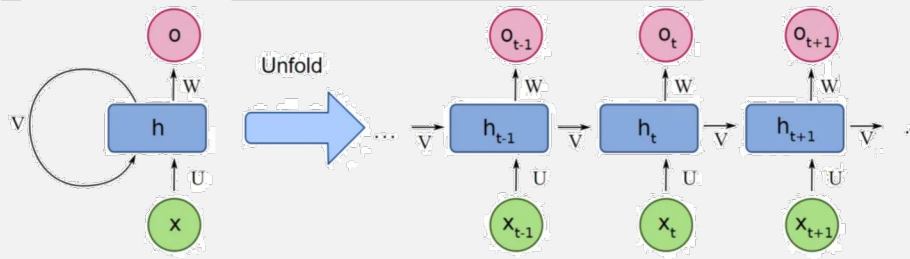
### Disadvantages

- Fixed context size makes it hard to use distant contexts


### Question

- Can a neural language model take advantage of information provided by the entire context – no matter how long it is?

## Recurrent neural network (RNN) language models




- RNN has recurrent connections from the hidden layer to itself
- Input tokens are provided sequentially
- The hidden layer accumulates contextual information over time
- Output at each time is informed by the context seen so far



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## Recurrent neural network (RNN) language models

**output distribution**

$$\hat{y} = \text{softmax}(W_2 h^{(t)})$$

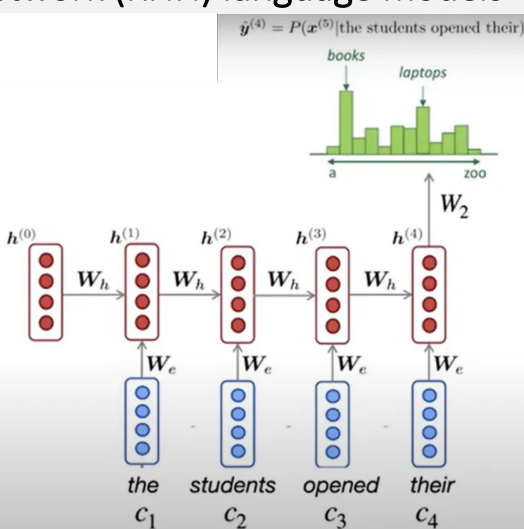
**hidden states**

$$h^{(t)} = f(W_h h^{(t-1)} + W_e c_t)$$

$h^{(0)}$  is initial hidden state!

**word embeddings**

$c_1, c_2, c_3, c_4$



The diagram illustrates an unrolled Recurrent Neural Network (RNN) for the sentence "the students opened their".

- Word Embeddings:** The words "the", "students", "opened", and "their" are represented by embeddings  $c_1, c_2, c_3, c_4$  (blue boxes).
- Hidden States:** The hidden states  $h^{(0)}, h^{(1)}, h^{(2)}, h^{(3)}, h^{(4)}$  (red boxes) are updated sequentially. The transition from  $h^{(t-1)}$  to  $h^{(t)}$  is governed by the hidden state weight matrix  $W_h$ . The embedding  $c_t$  is projected into the hidden state space using the embedding weight matrix  $W_e$ .
- Output Distribution:** The final hidden state  $h^{(4)}$  is passed through the output weight matrix  $W_2$  to produce the output distribution  $\hat{y}^{(4)} = P(x^{(5)} | \text{the students opened their})$ . The distribution shows peaks for "books" and "laptops", indicating the model's prediction for the next word.



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## Recurrent neural network language models

### Advantages

- No need to store large numbers of  $n$ -grams
- Can cope with data sparsity through generalization across semantically similar words and contexts
- The relevant probabilities are estimated by a neural network
- Can accommodate distant context to some extent

### Disadvantage

- Over time, the effect of words seen early in the sequence gets overridden by more recently seen words
- Basic RNN language models cannot accommodate distant context

## What Simple RNN Language Models can handle

- Easy case:
  - The sky is ....
  - The sky is blue
  - Easy because relevant context is near where it is needed

## What Simple RNN Language Models can't handle

- **Difficult case:**

- I was born in India. I studied Electrical Engineering as an undergraduate. I came to the United States for graduate studies. After a short stint at Drexel where I got my masters degree in Electrical and Computer Engineering, I moved to the University of Wisconsin for my PhD. My PhD research was on neural networks. I play tennis. I speak fluent ....
- Nearby context suggests the next word must be a language
- But which language?
- **The relevant context for accurately predicting the next word is very far away, well beyond the reach of the Markov assumption**

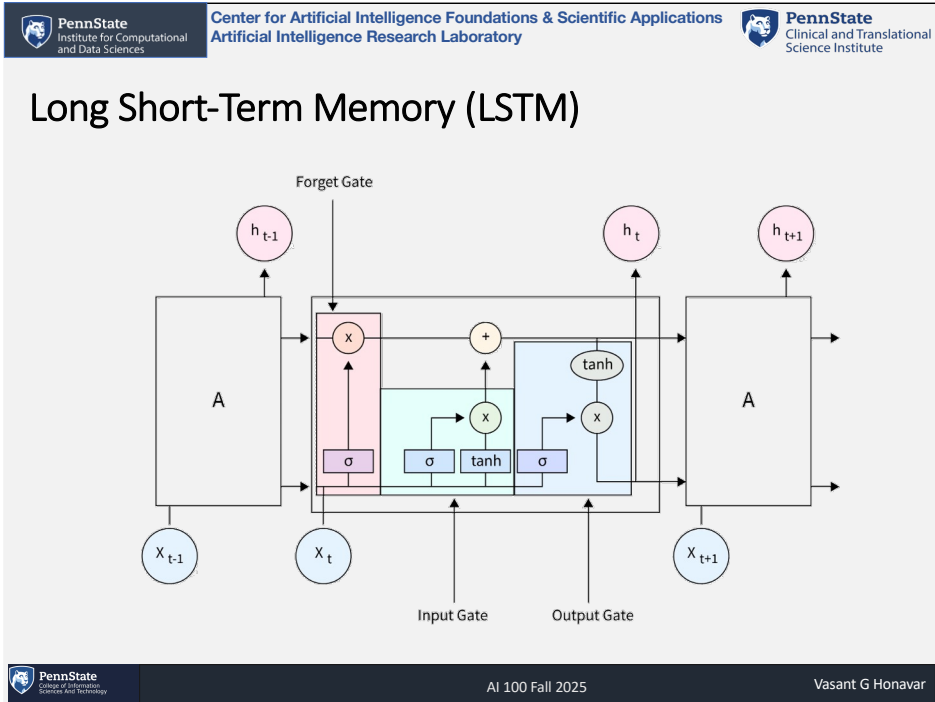
## LSTM Network Language Models

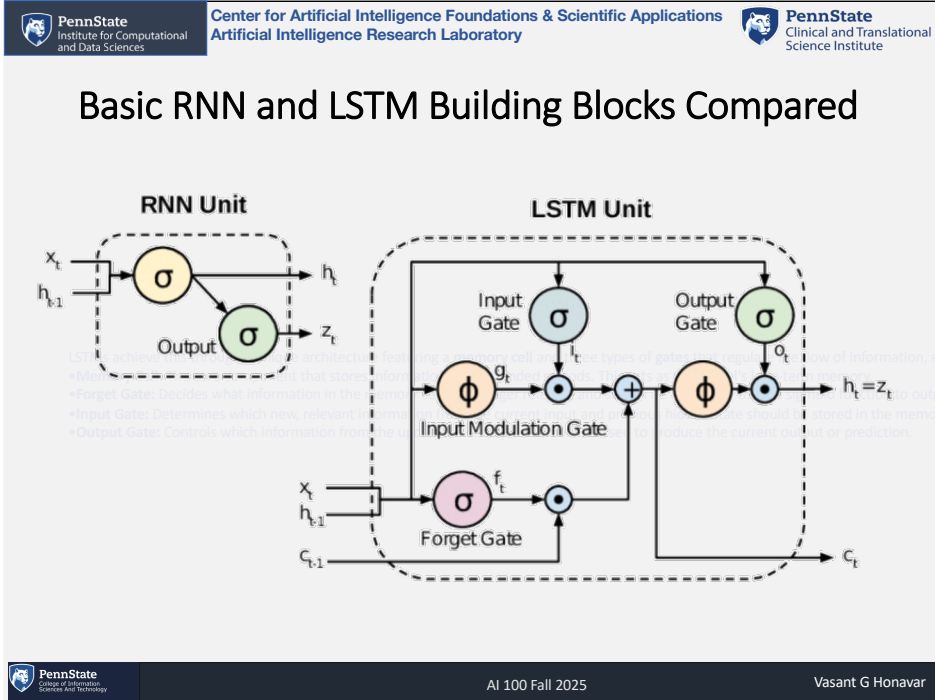
- An LSTM (Long Short-Term Memory) language model is a recurrent neural network (RNN) augmented with memory (LSTM)
- LSTM can process sequential data (like words in a sentence)
- LSTM can capture long-range contextual information
- Until the advent of transformers, LSTM networks were the state of the art for
  - Language generation
  - Language translation
  - Speech recognition




## How LSTM Network Language Models Work

- LSTM networks augment RNN with a memory cell and three types of gates that learn to regulate memory and help LSTM take advantage of long-range contextual information.
- **Memory Cell:** stores information over extended periods (long-term memory)
- **Forget Gate:** Decides what information in the memory cell is no longer relevant and should be discarded
- **Input Gate:** Determines which new, relevant information from the current input and previous hidden state should be stored in the memory cell.
- **Output Gate:** Controls which information from the updated memory cell should be used to produce the current output or prediction








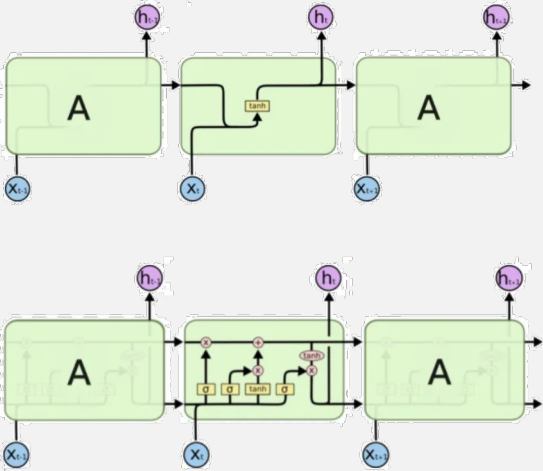
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## RNN and LSTM Network Architectures Compared





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## LSTM Network language models

### Advantages

- No need to store large numbers of  $n$ -grams
- Can cope with data sparsity through generalization across semantically similar words and contexts
- The relevant probabilities are estimated by a neural network
- Can accommodate distant context using LSTM

### Disadvantages


- Limited scalability to long sequences
- Limited scalability to large data sets

## Pre-trained Transformer-based LLM

- **Unidirectional models** (for example, the GPT family of models), which predict tokens from left to right using the transformer decoder.
- **Bidirectional models** (for example, BERT, RoBERTa), which contextualize each token based on both left and right contexts using the transformer encoder.
- **Sequence-to-sequence models** (for example, T5, BART), which combine an encoder and a decoder to support conditional text generation.
- Transformer plays a key role in all three


## Transformer is a sequence-to-sequence model





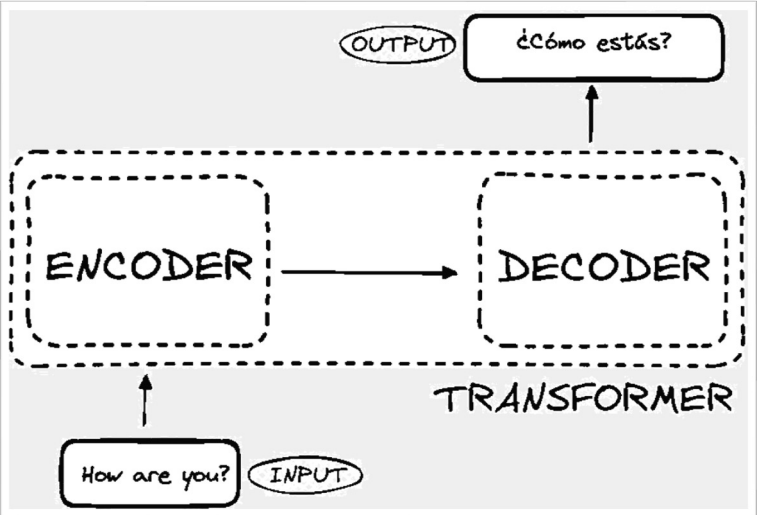
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## Transformer is an encoder-decoder model



```
graph LR; Input[How are you? INPUT] --> Encoder[ENCODER]; subgraph TRANSFORMER; Encoder --> Decoder[DECODER]; end; Decoder --> Output[OUTPUT: ¿Cómo estás?]
```



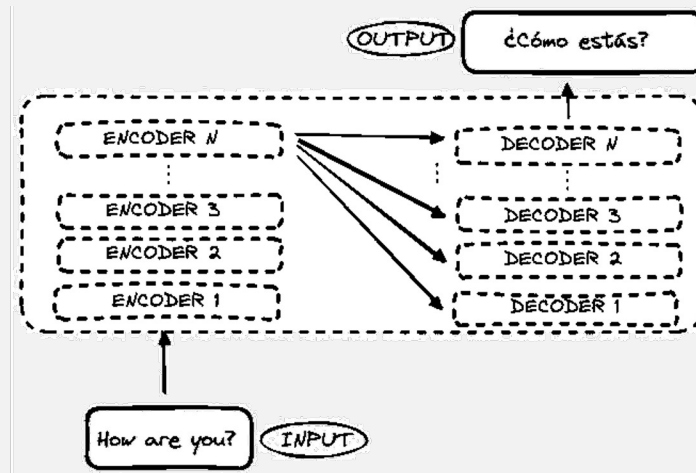
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## Transformer can stack encoders and decoders



## Transformer processes all tokens in parallel

- **Parallel processing makes transformers scalable** - Unlike RNN and LSTM which process words in a sentence sequentially, transformer reads all of the words simultaneously in parallel
- **Self attention**
  - Allows unlimited context – every word can interact with every other word simultaneously
  - Computes simultaneously for each word, a weighted combination of embeddings of all other words
  - Weights which model the contextual relevance of each word for all other words are learned from large text corpus

## How Attention works in a Transformer

### Example sentence

*The book that you gave me yesterday was fascinating.*

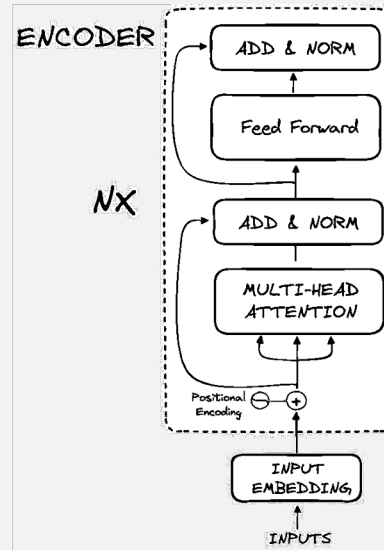
- To interpret the word **fascinating**, a model should pay special attention to the word **book**, which is the subject of the sentence.
- Self-attention learns to do this automatically:
  - the representation of **fascinating** will place high weight on **book** and lower weight on less relevant words like **yesterday**
  - The representation of **book** will place a high weight on **fascinating**
- Because self-attention directly connects every pair of words **each word can utilize useful context supplied all other words**, including those that are far away along the sequence.

## Multi-headed attention

- Transformers use multiple attention heads working in parallel.
- Each head can learn to specialize in a different type of linguistic pattern:
  - Examples subject–verb relationships
  - pronouns and their antecedents (co-reference)
  - modifier–noun relations (e.g., “red car”)
  - positional or structural patterns
- The results are aggregated

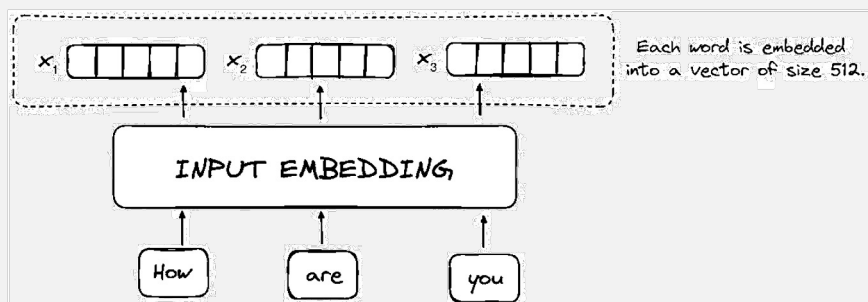
## Architecture of Encoders

- Encoder maps embeddings of words in the sentence to an internal representation
- The representation is shaped by attention
- Decoder decodes it



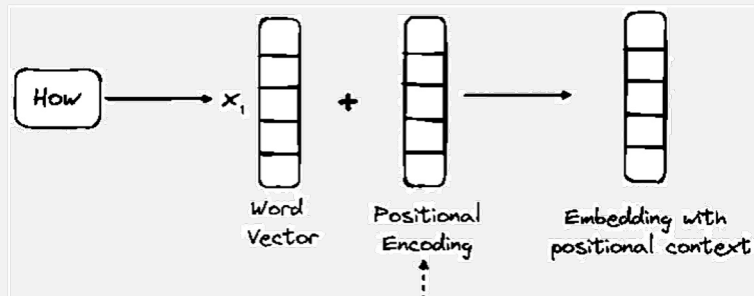
## Transformer in action: Embeddings

- Transformer starts by mapping words into their embeddings



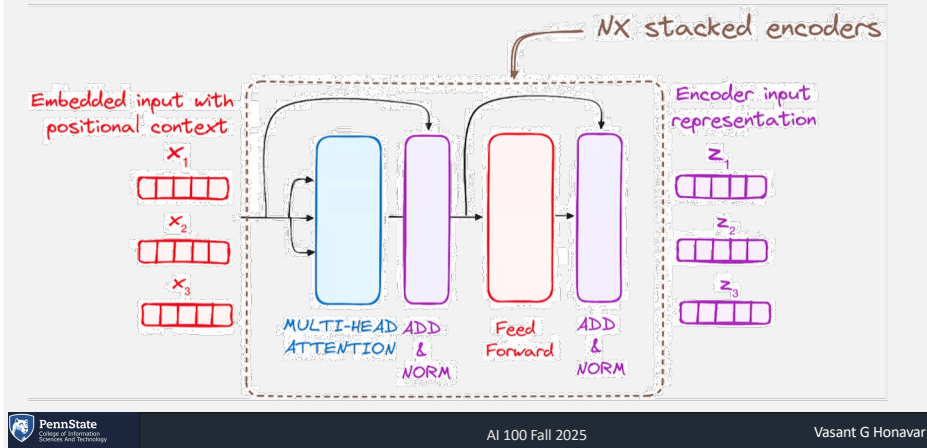
## Transformer in action: Positional encoding

- Transformer adds positional encoding to keep track of the position of each word in the sequence



## Transformer in action: Encoder stack

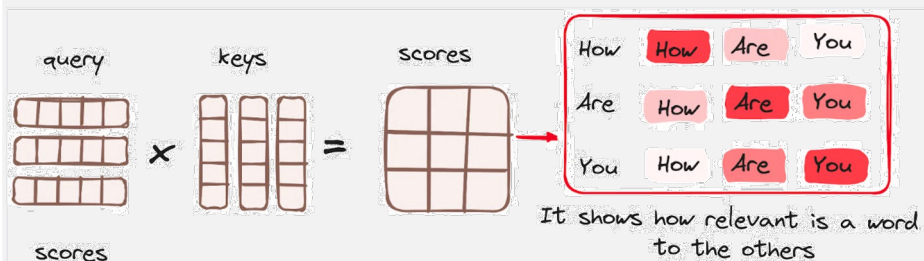
- The encoder layer transforms input sequence into a numeric abstract representation of the learned information from the entire sequence using multi-headed attention and linear weighting





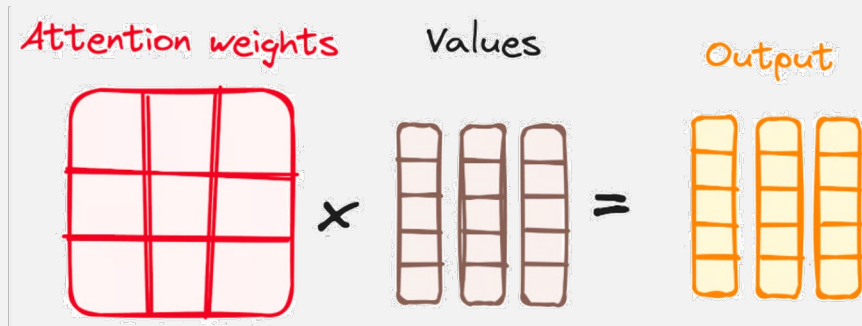
## Transformer in action: How attention works

- Transformer maps the word at each position to three vectors using learned weights: query, key, and value
- Attention of word  $i$  to word  $j$  is based on the similarity of the corresponding query and key
- Representation of a word is the attention weighted values of all other words



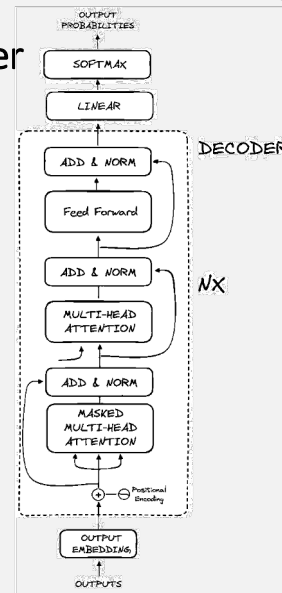
## Transformer in action: Encoder outputs

- Encoder output is a set of high scoring vectors that encode words
- These become inputs to the decoder



## Transformer in action: Decoder

- The decoder's role is to turn encoder output back into sentences.
- The structure of the decoder mirrors that of the encoder, in opposite order

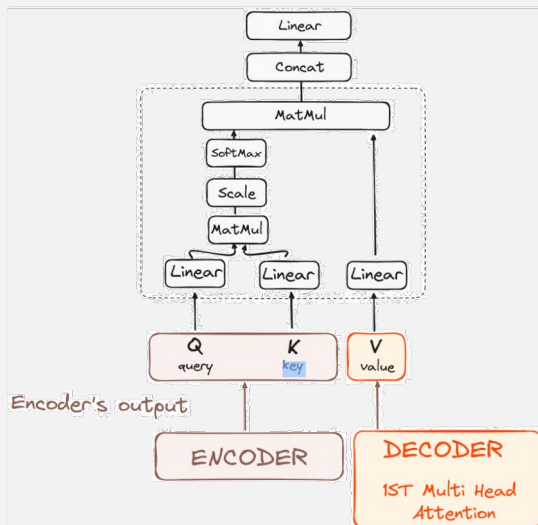


## Transformer in action: Masked Self-Attention

- Masked self-attention is like self-attention, but considers only words that have been generated so far – you can't look at words ahead at positions are yet to be generated in generating the current word

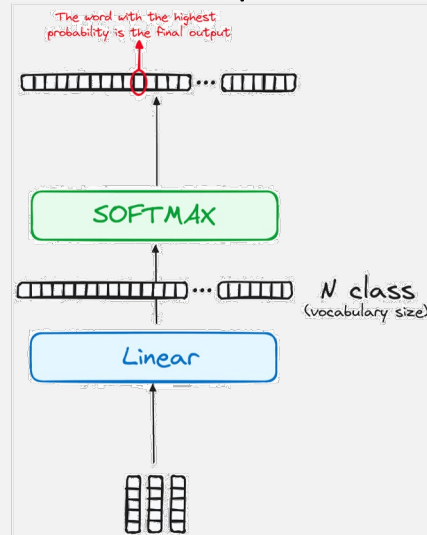
$$\begin{array}{|c|c|c|} \hline \text{scaled scores} & & \\ \hline 0.5 & 0.2 & 0.1 \\ \hline 0.1 & 0.6 & 0.2 \\ \hline 0.1 & 0.2 & 0.3 \\ \hline \end{array}
 + 
 \begin{array}{|c|c|c|} \hline \text{Look-ahead mask} & & \\ \hline 0 & -\text{inf} & -\text{inf} \\ \hline 0 & 0 & -\text{inf} \\ \hline 0 & 0 & 0 \\ \hline \end{array}
 = 
 \begin{array}{|c|c|c|} \hline \text{Masked Scores} & & \\ \hline 0.5 & -\text{inf} & -\text{inf} \\ \hline 0.1 & 0.6 & -\text{inf} \\ \hline 0.1 & 0.2 & 0.3 \\ \hline \end{array}$$

## Transformer in action: Align decoder outputs with inputs



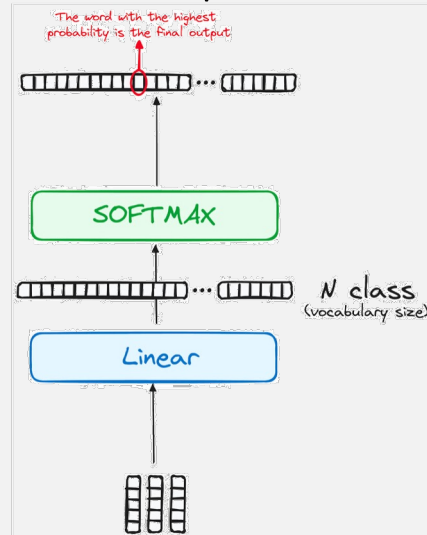
## Transformer in action: Generate word probabilities

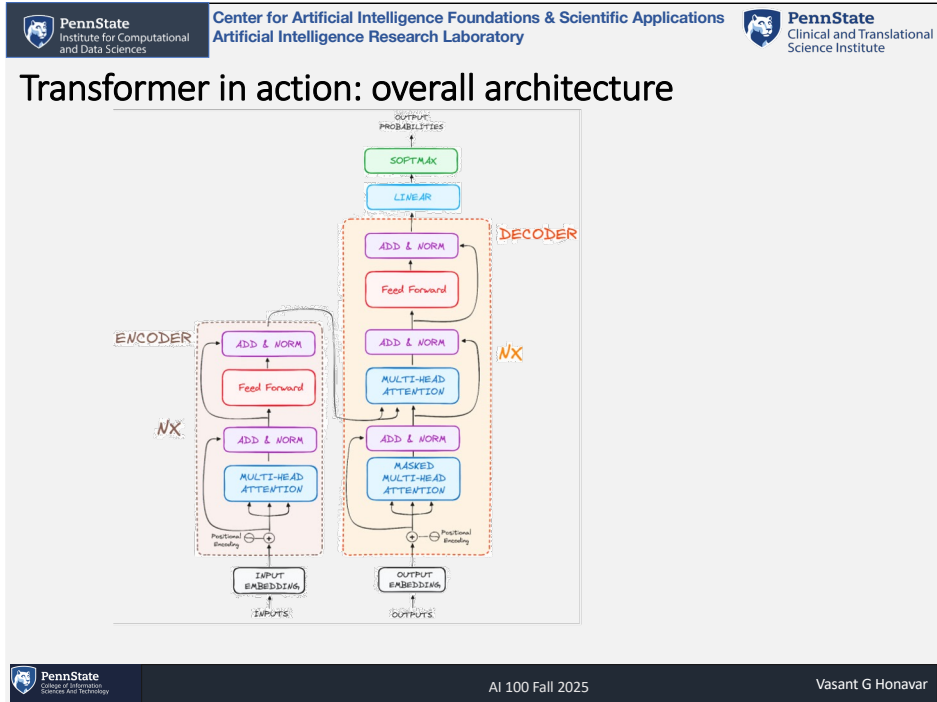
- Transformer generates probability of each word at each position in the sequence



## Transformer in action: Generate output sentence

- Transformer generates probability of each word at each position in the sequence
- Sample words according to their probability and produce the output sentence







## Kinds of transformer-based LLMs

- **Encoder-only models** (e.g., BERT, RoBERTa) suited for classification and language understanding.
- **Decoder-only models** (e.g., GPT, GPT-2, GPT-3, and their successors), optimized for text generation.
- **Encoder-decoder models** (e.g., T5, BART), for translation, summarization, and general sequence-to-sequence tasks.

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## Information flow in representative LLMs

GPT: Generation

BERT: Summarization

Source: Hang Li. 2022. ACM 65, 7, 56–63. <https://doi.org/10.1145/3490443>

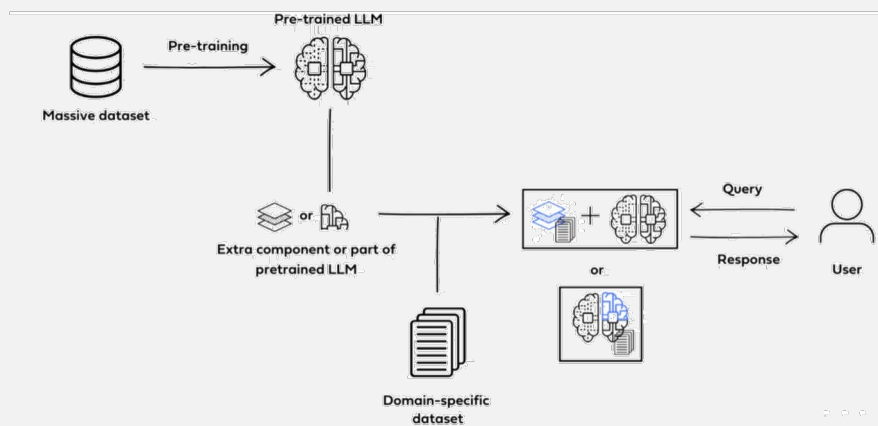
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## LLM Finetuning

- Pretrained models are often finetuned for specific tasks



## LLM Capabilities and Limitations

- LLM display remarkable language competence without language comprehension
  - LLM does not know what it means to be happy or sad, but can write poetry that expresses happiness or sadness
  - LLM can summarize text without understanding text
  - LLM can answer questions without understanding questions or answers
- LLMs cannot perform logical or mathematical reasoning
  - When LLM appears to do math or logic it is doing statistical next token prediction

## LLM Capabilities and Limitations

- **Hallucination**
  - LLM make up stuff, and sound confident when they do
  - Making stuff up may be fine in writing novels, but not in scientific writing or journalism
- **Bias, prejudice, misinformation, potential for harm**
  - LLM learn from human-generated text
- **Difficulty of evaluation**
  - Information leakage
- **Model collapse**
  - When LLM get trained on LLM-generated data then what?
- **Lack of alignment with human values and ethics**
- **Environmental impact**

## Future possibilities

- Multi-modal models
  - Vision-language
- Physically embodied models
  - Can learn richer semantics
- Generative-Deliberative hybrids
  - LLM guesses, theorem-prover or experiment verifies