ARTIFICIAL INTELLIGENCE

The Very Idea

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Problem solving through problem decomposition

- Problem
  - Solving an integral
- Sub-problems
  - Easier integrals to solve
- Actions or operators
  - Rules of integral calculus and algebra
- Primitive problems
  - Problems whose solutions can be looked up or computed by executing a known procedure

Example

- Problem
  - solving an integral
- Sub-problems
  - easier integrals to solve
- Operators
  - rules of integral calculus and algebra
- Primitive problems
  - problems whose solutions can be looked up or computed by executing a known procedure

$$\int (2t + sec^2t\ sin^2t + 7\ )dt$$

AND

$$\int 2t\ dt + \int sec^2t\ sin^2t\ dt + \int 7\ dt$$

OR

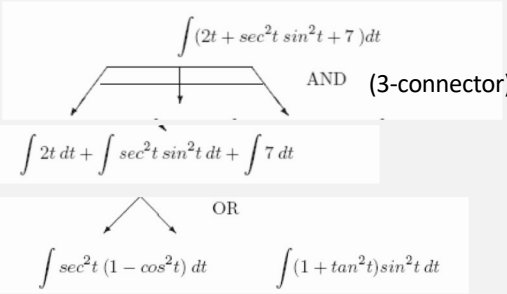
$$\int sec^2t\ (1 - cos^2t)\ dt \qquad \int (1 + tan^2t) sin^2t\ dt$$

Problem reduction representation (PRR)

- A PRR problem is specified by a 3-tuple (*G, O, P*)
  - *G* is a problem to be solved
  - *O* is a set of operators for decomposing problems into sub-problems through AND or OR decompositions
  - *P* is a set of primitive problems with known solutions
- Solution
  - An AND decomposition is solved when each of the sub-problems is solved
  - An OR decomposition is solved when at least one of the sub-problems is solved
  - A problem is unsolvable if it is neither a primitive problem nor can it be further decomposed
- PRR is a generalization of the state space representation (why?)

Problem reduction representation

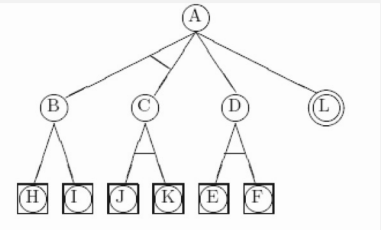
- Solving a problem in PRR reduces to searching an AND-OR graph
- Nodes correspond to problems
- Connectors correspond to actions
- Connectors correspond to AND or OR decompositions
- Connectors of arity *k* are called *k*-connectors

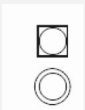


(3-connector)

Example

Problem

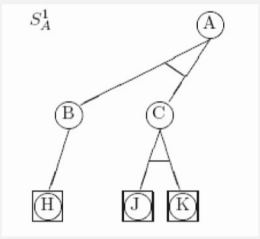


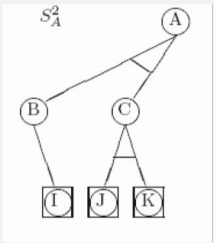


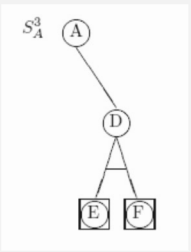
Primitive problem

Unsolvable problem

3 solutions







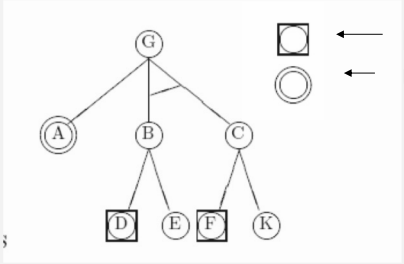
Solution to an PRR problem

- A sub-graph s_q of an AND-OR graph is said to be solution to a problem q if
  - s_q is rooted at q
  - Each non-leaf node y in s_q, has exactly one connector out of it that belongs to s_q
  - Each leaf node in s_q is a primitive problem (i.e. a member of P)
- A problem q is said to be solvable if
  - a sub-graph s_q of an AND-OR graph is a solution to q
- Solving a problem G using a PRR (G, O, P) entails finding a sub-graph S_G of the corresponding AND-OR graph that is a solution of G

Question – How can we solve an PRR problem?

- Basic idea:
  - Generalize state-space search
- How?
  - Generalize partial paths to sub graphs of the PRR AND-OR graph
  - Expanding a node must comply with the semantics of AND and OR connectors
  - Termination test must comply with the definition of a solution

Example – BFS



Primitive problem

Unsolvable problem

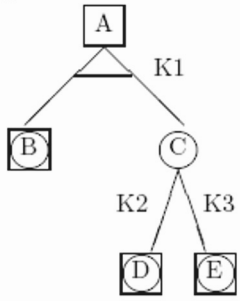
List

(G)
(A) (B & C)
(B & C)
(D & F) (E & F) (D & K) (E & K)

Exercise: Solve the same problem using DFS

Optimal (minimum cost) solution of AND-OR graphs

Example:

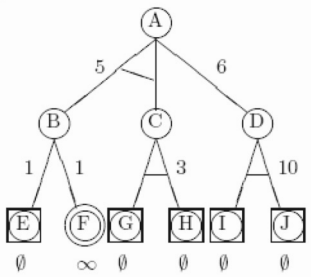


- Cost of an unsolvable primitive problem – infinity
- Cost of connectors and primitive problems are assumed to be strictly positive and bounded

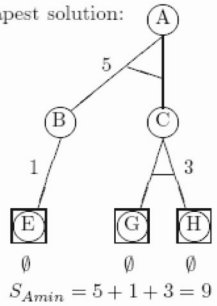
$$Cost(S_A) = Cost(k_1) + Cost(S_B) + Cost(S_C)$$

Optimal solution of an SRR problem

Example:

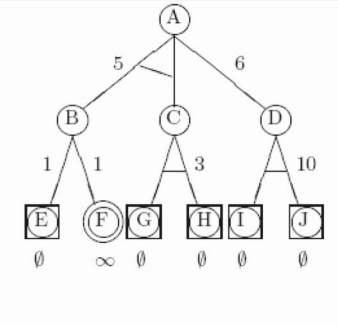


Cheapest solution:



$S_{Amin} = 5 + 1 + 3 = 9$

Branch and Bound Search for Optimal Solution



List

(A, 0)

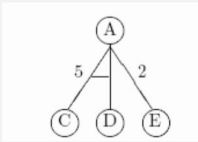
(B & C, 5) (D, 6)

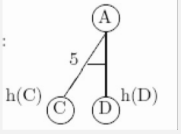
(E & C, 6)(D, 6) (F & C, ∞)

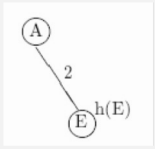
(D, 6) (E & G & H, 9) (F & C, ∞)

(E & G & H, 9) (I & J, 16) (F & C, ∞)

Using Heuristics







$$f(C\&D) = 5 + h(C) + h(D)$$

$$f(E) = 2 + h(E)$$

Admissible heuristic function

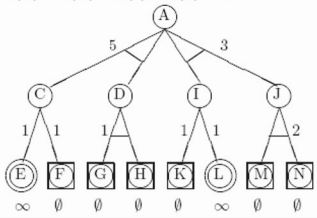
$h(n) \le h^*(n) = Cost(n)$ ← Cost of the cheapest solution of *n*

AO* - Searching AND-OR graphs

Example:

$h(A) = 0$

$h(C) = h(D) = h(I) = h(J) = 1$



(A, 0+0)

(I & J, 3+2) (C & D, 5+2)

(K & M & N,6) (C & D, 5+2) (L & M & N, ∞)

Properties of AO*

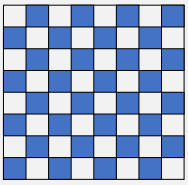
- AO* is a generalization of A* for AND-OR graphs
- AO*, like A*, is admissible if the heuristic function is admissible under the usual assumptions
  - finite branching factor
  - strictly positive action costs
- AO*, like A* is also optimal among the class of heuristic search algorithms that use an additive cost evaluation function

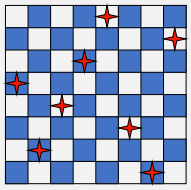
Solving Constraint Satisfaction Problems

Constraint Satisfaction Problem (CSP)

- Special class of state-space search problems
- States are defined by values assigned to some or all of the variables $\{X_1, X_2, \ldots, X_n\}$
  - Each variable X_i takes values from a domain D_i
  - In the most common setting, D_i is finite
- The assignment of values to variables is subject to a set of constraints $\{C_1, C_2, \ldots, Cp\}$
  - Each constraint relates a subset of variables by specifying the valid combinations of their values
- Solution
  - A goal state in which every variable has a value assigned to it and the assignment satisfies the specified constraints
  - Note: We don't care about the path from start state to goal state, only the assignment of variables in the goal state

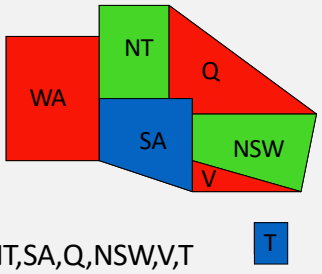
Example: Constraint satisfaction problem





- Variables: rows $Q_1 \cdots Q_8$
- Domain of each variable represents column in which a queen is placed : $\{1 \cdots 8\}$
- Constraints
  - for all $i < j$
    - $Q_i \neq Q_j$ (queens cannot share a row)
    - $|Q_i - Q_j| \neq |i - j|$ (no two queens can share a diagonal)
- Solution
  - An assignment of values to $Q_1 \cdots Q_8$ that satisfies the constraints

Example: Map Coloring Problem



Western Australia
Northern Territory
Queensland
South Australia
New South Wales
Victoria
Tasmania

- Variables: WA,NT,SA,Q,NSW,V,T
- Domain of each variable: {red, green, blue}
- Constraints: No two adjacent variables can have the same value:

  WA≠NT, WA≠SA, NT≠SA, NT≠Q, SA≠Q,
  SA≠NSW, SA≠V, Q≠NSW, NSW≠V

Example: Course scheduling

- Variables: Courses
- Domains of variables: Cross product of rooms, timeslots and instructors
- Constraints:
  - No two courses can share the same instructor and time slot
  - No two courses can share the same room and time slot
- Solution: An assignment of courses to rooms, timeslots and instructors that satisfies the constraints

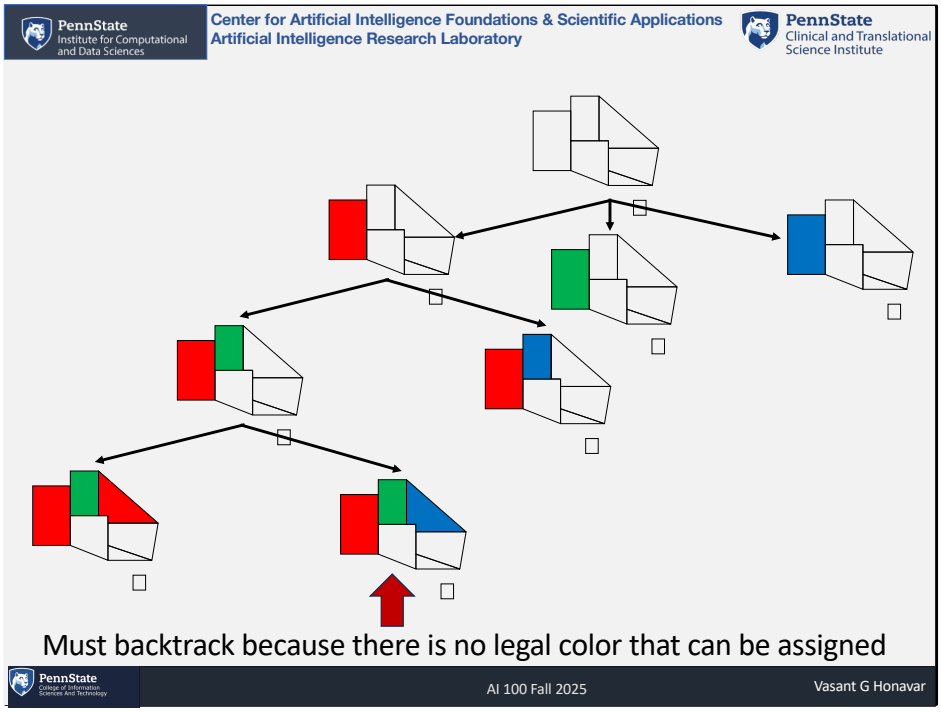
CSP as a Search Problem

- n variables $X_1, \ldots, X_n$
- Valid assignment: Assignment of values to the variables without violating any of the constraints
- Complete assignment: one where each variable has a value
- States: valid assignment (no violation of constraints)
- Initial state: empty assignment
- Successor of a state: assign a value to a variable without a value assignment
- Goal test: complete assignment (which by design is also a valid assignment)

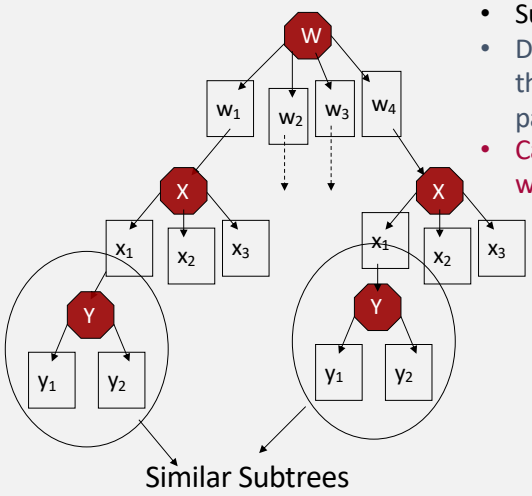
Key properties of (Discrete) CSP

- Search space is finite
- The order in which variables are assigned values has no impact on the reachable complete valid assignments
  - One can expand a node by first selecting any unassigned variable and assign it a value from its domain without violating the constraints
  - Big reduction in branching factor compared to standard search
- The solution is always of fixed depth = number of variables
- The path from start state to goal state does not matter
- All we care about is satisfying the constraints on the goal
  - We can use a simplified depth-first search with backtracking

Example: Backtracking search



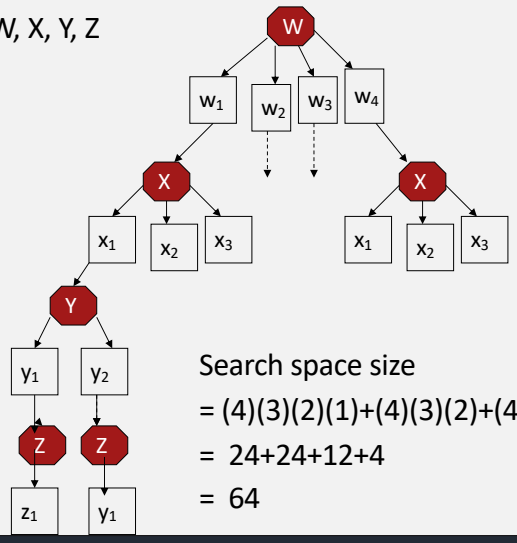
Sub-trees have similar topologies



- Suppose incompatible with $Y = y_1$
- Depth first search will rediscover this incompatibility in different parts of the state space
- Can we avoid this unnecessary work?

Variable ordering impacts size of search space

Variable ordering W, X, Y, Z



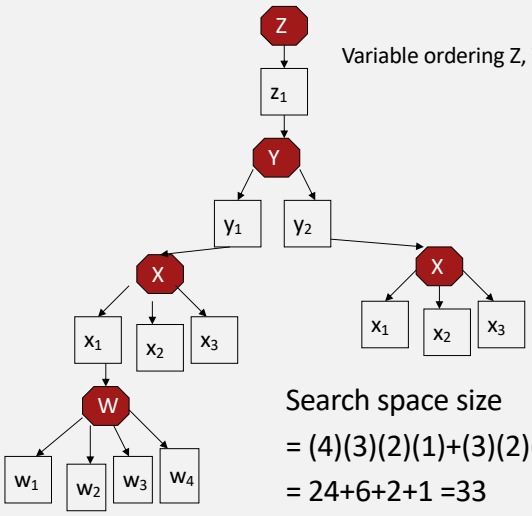
Search space size

= (4)(3)(2)(1)+(4)(3)(2)+(4)(3)+(4)

= 24+24+12+4

= 64

Variable ordering and Search space size



Variable ordering Z, Y, Z, W

Search space size

= (4)(3)(2)(1)+(3)(2)(1)+(2)(1)+1

= 24+6+2+1 =33

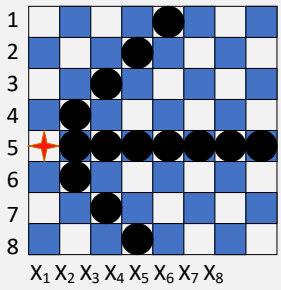
Backtracking search

- Standard backtracking fails to exploit special properties of CSP
  - Subtrees have similar topologies
    - Can we eliminate the duplicate work of rediscovering incompatible assignments?
    - Yes, if we can propagate constraints
  - Search space has minimal size under a certain ordering of variables (most constrained to least constrained)
    - Can we consider the variables and value assignments in some order that effectively minimizes the size of the search space to be considered?

Propagating constraints: Forward checking

Avoiding rediscovering incompatible assignments

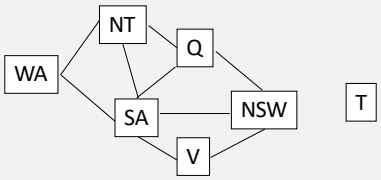
- Propagate the constraints



Assigning the value 5 to X_1 leads to removing values from the domains of $X_2, X_3, \cdots X_8$

Constraint propagation through forward checking

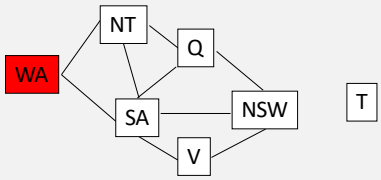
Constraint graph facilitates constraint propagation



WA	NT	Q	NSW	V	SA	T
RGB	RGB	RGB	RGB	RGB	RGB	RGB

Constraint propagation through forward checking

Constraint graph facilitates constraint propagation

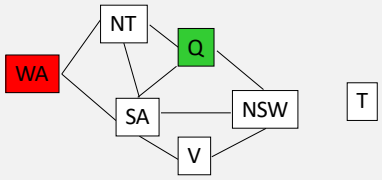


WA	NT	Q	NSW	V	SA	T
RGB	~~R~~GB	RGB	RGB	RGB	~~R~~GB	RGB
R	GB	RGB	RGB	RGB	GB	RGB

Forward checking removes the value Red of NT and of SA

Constraint propagation through forward checking

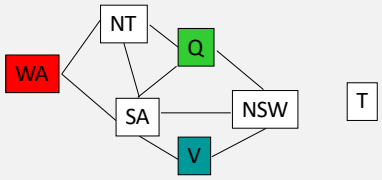
Constraint graph facilitates constraint propagation



WA	NT	Q	NSW	V	SA	T
RGB	RGB	RGB	RGB	RGB	RGB	RGB
R	GB	RGB	RGB	RGB	GB	RGB
R	~~G~~B	G	R~~G~~B	RGB	~~G~~B	RGB

Constraint propagation through forward checking

Constraint graph facilitates constraint propagation



WA	NT	Q	NSW	V	SA	T
RGB	RGB	RGB	RGB	RGB	RGB	RGB
R	GB	RGB	RGB	RGB	GB	RGB
R	B	G	RB	RGB	B	RGB
R	B	G	~~RB~~	B	~~B~~	RGB

Constraint propagation through forward checking

Constraint graph facilitates constraint propagation

Empty set: the current assignment {(WA ← R), (Q ← G), (V ← B)} does not lead to a solution

WA	NT	Q	NSW	V	SA	T
RGB	RGB	RGB	RGB	RGB	RGB	RGB
R	GB	RGB	RGB	RGB	GB	RGB
R	B	G	RB	RGB	B	RGB
R	B	G	~~RB~~	B	~~B~~	RGB

Constraint propagation through Forward Checking

Whenever a pair $(X \leftarrow v)$ is added to assignment do:

For each variable Y not in the assignment:

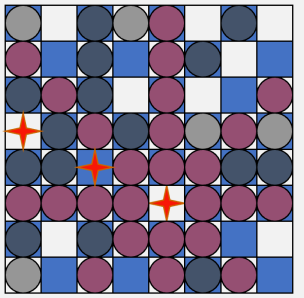
For every constraint C relating Y to the variables in the assignment:

Remove all values from Y's domain that do not satisfy C

Modified Backtracking Algorithm

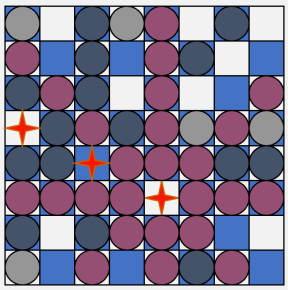
- Recall that search space is minimized by appropriate choice of the order in which variables and values are considered
- Which variable X_i should be assigned a value next?
  - Most-constrained-variable heuristic
    - Consider the variable with the fewest values in its domain
  - Most-constraining-variable heuristic
    - Consider the variable that will constrain the values of the largest number of variables
- In which order should its values be assigned?
  - Least-constraining-value heuristic
    - Assign the value that allows the greatest flexibility in assigning values to the remaining variables

8-Queens



4 3 2 3 4 ← Numbers of values for each un-assigned variable

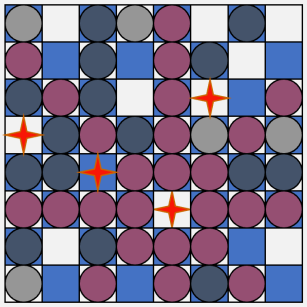
8-Queens



4 3 2 3 4 ← Numbers of values for each un-assigned variable

New assignment

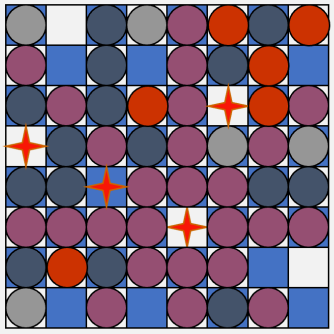
8-Queens



4 3 2 3 4 ← Numbers of values for each un-assigned variable

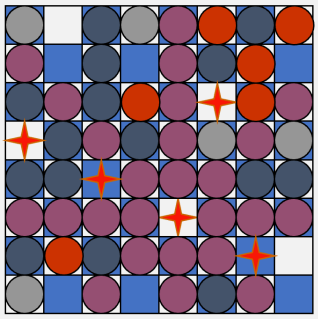
New assignment

8-Queens



3 2 1 3 ← New numbers of values for each un-assigned variable

8-Queens

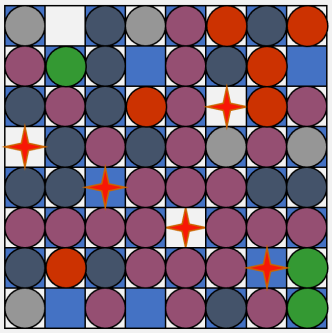


New assignment

3 2 1 3

New numbers of values for each un-assigned variable

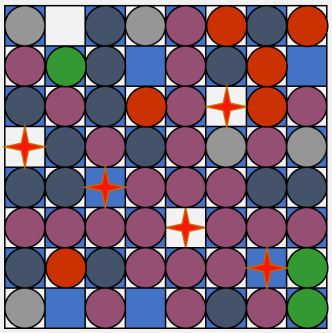
8-Queens



Forward checking

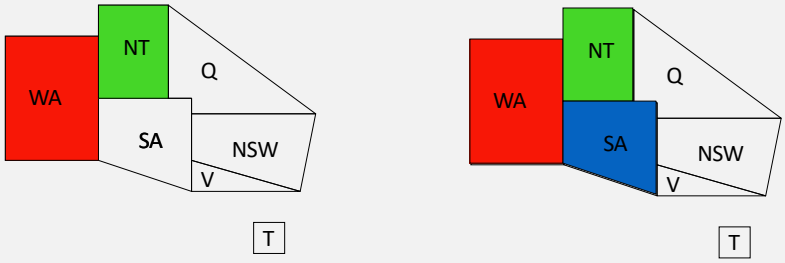
New assignment

8-Queens



--------- Next placement

Map Coloring



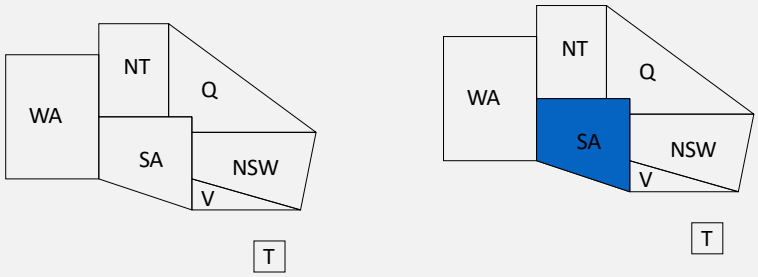
- SA's remaining domain has size 1 (value Blue remaining)
- Q's remaining domain has size 2
- NSW's, V's, and T's remaining domains have size 3

Hence, we select SA

Most-Constraining-Variable Heuristic

- Which variable *X* should be assigned a value next?
- Among the variables with the smallest remaining domains (ties with respect to the most-constrained-variable heuristic)
  - select the one that appears in the largest number of constraints on variables not in the current assignment
- Rationale: Increase future elimination of values, to reduce future branching factors

Map Coloring



- Before any value has been assigned, all variables have a domain of size 3, but SA is involved in more constraints (5) than any other variable

Hence, select SA and assign a value to it (e.g., Blue)

Least-Constraining-Value Heuristic

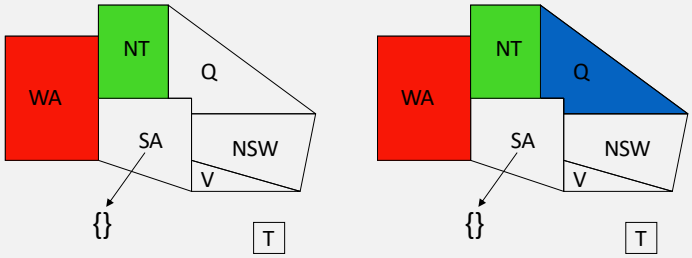
- In which order should X's values be assigned?

  Select the value of X that removes the smallest number of values from the domains of those variables which are not in the current assignment

  Rationale: Since only one value will eventually be assigned to X, pick the least-constraining value first, since it is the most likely not to lead to an invalid assignment

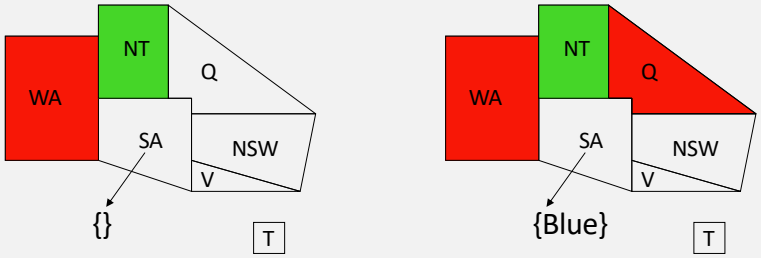
  Note: Using this heuristic requires performing a forward-checking step for every value, not just for the selected value

Map Coloring



- Q's domain has two remaining values: Blue and Red
- Assigning Blue to Q would leave 0 value for SA, while assigning Red would leave 1 value

Map Coloring



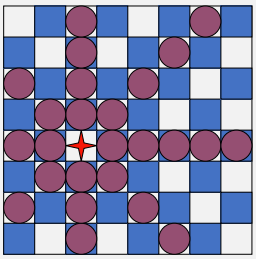
- Q's domain has two remaining values: Blue and Red
- Assigning Blue to Q would leave 0 value for SA, while assigning Red would leave 1 value

Hence, assign Red to Q

Applications of CSP

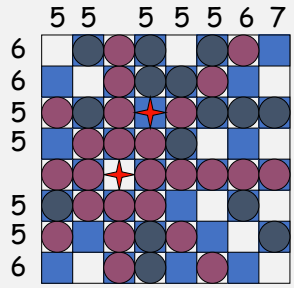
- CSP techniques are widely used
- Applications include:
  - Course scheduling
  - Crew assignments to flights
  - Management of transportation fleet
  - Flight/rail schedules
  - Job shop scheduling
  - Task scheduling in port operations
  - Design, including spatial layout design

Constraint Propagation



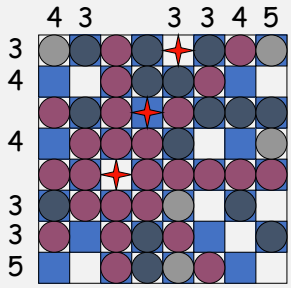
- Place a queen in a square
- Remove the attacked squares from future consideration

Constraint Propagation



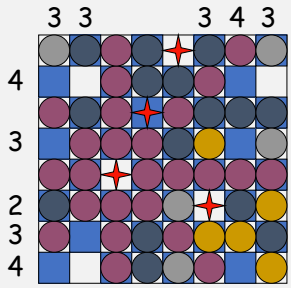
- Count the number of non-attacked squares in every row and column
- Place a queen in a row or column with minimum number
- Remove the attacked squares from future consideration

Constraint Propagation

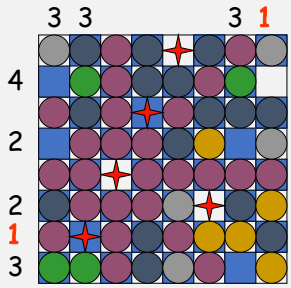


- Repeat

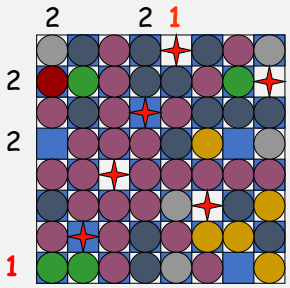
Constraint Propagation



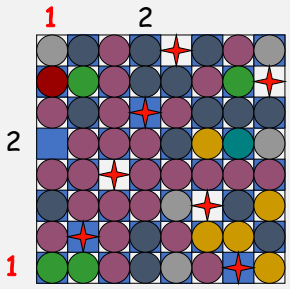
Constraint Propagation



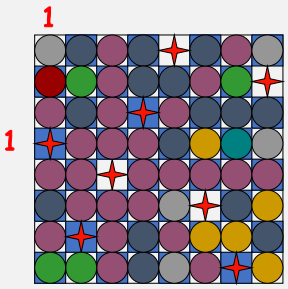
Constraint Propagation



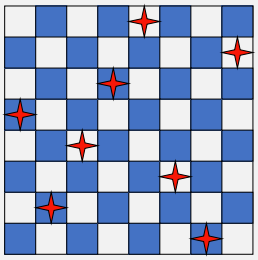
Constraint Propagation



Constraint Propagation



Constraint Propagation



Constraint propagation

Constraint propagation finds broad applications

- Scheduling
  - Surgeries
  - Jobs to machines
  - Courses
  - Rooms
- Map coloring
- Layout Problems
  - Road layout
  - VLSI layout
  - Floor plan layout
- 3D interpretation of 2D drawings
- ….